c) Let $S(x,y)$ be "System x is in state y." Recalling that "only if" indicates a necessary condition, we have $S(\text{firewall, diagnostic}) \to S(\text{proxy server, diagnostic})$.
d) Let $T(x)$ be "The throughput is at least x kbps," where the domain of discourse is positive numbers, let $M(x,y)$ be "Resource x is in mode y," and let $S(x,y)$ be "Router x is in state y." Then we have $(T(100) \land \neg T(500) \land \neg M(\text{proxy server, diagnostic})) \to \exists x\, S(x, \text{normal})$.

44. We want propositional functions P and Q that are sometimes, but not always, true (so that the second biconditional is $\mathbf{F} \leftrightarrow \mathbf{F}$ and hence true), but such that there is an x making one true and the other false. For example, we can take $P(x)$ to mean that x is an even number (a multiple of 2) and $Q(x)$ to mean that x is a multiple of 3. Then an example like $x = 4$ or $x = 9$ shows that $\forall x(P(x) \leftrightarrow Q(x))$ is false.

46. **a)** There are two cases. If A is true, then $(\forall x P(x)) \lor A$ is true, and since $P(x) \lor A$ is true for all x, $\forall x(P(x) \lor A)$ is also true. Thus both sides of the logical equivalence are true (hence equivalent). Now suppose that A is false. If $P(x)$ is true for all x, then the left-hand side is true. Furthermore, the right-hand side is also true (since $P(x) \lor A$ is true for all x). On the other hand, if $P(x)$ is false for some x, then both sides are false. Therefore again the two sides are logically equivalent.
b) There are two cases. If A is true, then $(\exists x P(x)) \lor A$ is true, and since $P(x) \lor A$ is true for some (really all) x, $\exists x(P(x) \lor A)$ is also true. Thus both sides of the logical equivalence are true (hence equivalent). Now suppose that A is false. If $P(x)$ is true for at least one x, then the left-hand side is true. Furthermore, the right-hand side is also true (since $P(x) \lor A$ is true for that x). On the other hand, if $P(x)$ is false for all x, then both sides are false. Therefore again the two sides are logically equivalent.

48. **a)** There are two cases. If A is false, then both sides of the equivalence are true, because a conditional statement with a false hypothesis is true. If A is true, then $A \to P(x)$ is equivalent to $P(x)$ for each x, so the left-hand side is equivalent to $\forall x\, P(x)$, which is equivalent to the right-hand side.
b) There are two cases. If A is false, then both sides of the equivalence are true, because a conditional statement with a false hypothesis is true (and we are assuming that the domain is nonempty). If A is true, then $A \to P(x)$ is equivalent to $P(x)$ for each x, so the left-hand side is equivalent to $\exists x\, P(x)$, which is equivalent to the right-hand side.

50. It is enough to find a counterexample. It is intuitively clear that the first proposition is asserting much more than the second. It is saying that one of the two predicates, P or Q, is universally true; whereas the second proposition is simply saying that for every x either $P(x)$ or $Q(x)$ holds, but which it is may well depend on x. As a simple counterexample, let $P(x)$ be the statement that x is odd, and let $Q(x)$ be the statement that x is even. Let the domain of discourse be the positive integers. The second proposition is true, since every positive integer is either odd or even. But the first proposition is false, since it is neither the case that all positive integers are odd nor the case that all of them are even.

52. **a)** This is false, since there are many values of x that make $x > 1$ true.
b) This is false, since there are two values of x that make $x^2 = 1$ true.
c) This is true, since by algebra we see that the unique solution to the equation is $x = 3$.
d) This is false, since there are no values of x that make $x = x + 1$ true.

54. There are only three cases in which $\exists!x P(x)$ is true, so we form the disjunction of these three cases. The answer is thus $(P(1) \land \neg P(2) \land \neg P(3)) \lor (\neg P(1) \land P(2) \land \neg P(3)) \lor (\neg P(1) \land \neg P(2) \land P(3))$.

56. A Prolog query returns a yes/no answer if there are no variables in the query, and it returns the values that make the query true if there are.

a) None of the facts was that Kevin was enrolled in EE 222. So the response is `no`.
b) One of the facts was that Kiko was enrolled in Math 273. So the response is `yes`.
c) Prolog returns the names of the courses for which Grossman is the instructor, namely just `cs301`.
d) Prolog returns the names of the instructor for CS 301, namely `grossman`.
e) Prolog returns the names of the instructors teaching any course that Kevin is enrolled in, namely `chan`, since Chan is the instructor in Math 273, the only course Kevin is enrolled in.

58. Following the idea and syntax of Example 28, we have the following rule:
```
grandfather(X,Y) :- father(X,Z), father(Z,Y); father(X,Z), mother(Z,Y).
```
Note that we used the comma to mean "and" and the semicolon to mean "or." For `X` to be the grandfather of `Y`, `X` must be either `Y`'s father's father or `Y`'s mother's father.

60. **a)** $\forall x(P(x) \to Q(x))$ **b)** $\exists x(R(x) \land \neg Q(x))$ **c)** $\exists x(R(x) \land \neg P(x))$
d) Yes. The unsatisfactory excuse guaranteed by part (b) cannot be a clear explanation by part (a).

62. **a)** $\forall x(P(x) \to \neg S(x))$ **b)** $\forall x(R(x) \to S(x))$ **c)** $\forall x(Q(x) \to P(x))$ **d)** $\forall x(Q(x) \to \neg R(x))$
e) Yes. If x is one of my poultry, then he is a duck (by part (c)), hence not willing to waltz (part (a)). Since officers are always willing to waltz (part (b)), x is not an officer.

SECTION 1.4 Nested Quantifiers

2. **a)** There exists a real number x such that for every real number y, $xy = y$. This is asserting the existence of a multiplicative identity for the real numbers, and the statement is true, since we can take $x = 1$.
b) For every real number x and real number y, if x is nonnegative and y is negative, then the difference $x - y$ is positive. Or, more simply, a nonnegative number minus a negative number is positive (which is true).
c) For every real number x and real number y, there exists a real number z such that $x = y + z$. This is a true statement, since we can take $z = x - y$ in each case.

4. **a)** Some student in your class has taken some computer science course.
b) There is a student in your class who has taken every computer science course.
c) Every student in your class has taken at least one computer science course.
d) There is a computer science course that every student in your class has taken.
e) Every computer science course has been taken by at least one student in your class.
f) Every student in your class has taken every computer science course.

6. **a)** Randy Goldberg is enrolled in CS 252.
b) Someone is enrolled in Math 695.
c) Carol Sitea is enrolled in some course.
d) Some student is enrolled simultaneously in Math 222 and CS 252.
e) There exist two distinct people, the second of whom is enrolled in every course that the first is enrolled in.
f) There exist two distinct people enrolled in exactly the same courses.

8. **a)** $\exists x \exists y Q(x,y)$
b) This is the negation of part (a), and so could be written either $\neg\exists x \exists y Q(x,y)$ or $\forall x \forall y \neg Q(x,y)$.
c) We assume from the wording that the statement means that the same person appeared on both shows: $\exists x(Q(x, \text{Jeopardy}) \land Q(x, \text{Wheel of Fortune}))$
d) $\forall y \exists x Q(x,y)$ **e)** $\exists x_1 \exists x_2(Q(x_1, \text{Jeopardy}) \land Q(x_2, \text{Jeopardy}) \land x_1 \neq x_2)$

10. a) $\forall x F(x, \text{Fred})$ b) $\forall y F(\text{Evelyn}, y)$ c) $\forall x \exists y F(x, y)$ d) $\neg\exists x \forall y F(x, y)$ e) $\forall y \exists x F(x, y)$
f) $\neg\exists x(F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$
g) $\exists y_1 \exists y_2(F(\text{Nancy}, y_1) \wedge F(\text{Nancy}, y_2) \wedge y_1 \neq y_2 \wedge \forall y(F(\text{Nancy}, y) \to (y = y_1 \vee y = y_2)))$
h) $\exists y(\forall x F(x, y) \wedge \forall z(\forall x F(x, z) \to z = y))$ i) $\neg\exists x F(x, x)$
j) $\exists x \exists y(x \neq y \wedge F(x, y) \wedge \forall z((F(x, z) \wedge z \neq x) \to z = y))$ (We do not assume that this sentence is asserting that this person can or cannot fool her/himself.)

12. The answers to this exercise are not unique; there are many ways of expressing the same propositions sym-
bolically. Note that $C(x, y)$ and $C(y, x)$ say the same thing.
a) $\neg I(\text{Jerry})$ b) $\neg C(\text{Rachel}, \text{Chelsea})$ c) $\neg C(\text{Jan}, \text{Sharon})$ d) $\neg\exists x\, C(x, \text{Bob})$
e) $\forall x(x \neq \text{Joseph} \leftrightarrow C(x, \text{Sanjay}))$ f) $\exists x \neg I(x)$ g) $\neg\forall x\, I(x)$ (same as (f))
h) $\exists x \forall y(x = y \leftrightarrow I(y))$ i) $\exists x \forall y(x \neq y \leftrightarrow I(y))$ j) $\forall x(I(x) \to \exists y(x \neq y \wedge C(x, y)))$
k) $\exists x(I(x) \wedge \forall y(x \neq y \to \neg C(x, y)))$ l) $\exists x \exists y(x \neq y \wedge \neg C(x, y))$ m) $\exists x \forall y\, C(x, y)$
n) $\exists x \exists y(x \neq y \wedge \forall z \neg(C(x, z) \wedge C(y, z)))$ o) $\exists x \exists y(x \neq y \wedge \forall z(C(x, z) \vee C(y, z)))$

14. The answers to this exercise are not unique; there are many ways of expressing the same propositions sym-
bolically. Our domain of discourse for persons here consists of people in this class. We need to make up a
predicate in each case.
a) Let $S(x, y)$ mean that person x can speak language y. Then our statement is $\exists x\, S(x, \text{Hindi})$.
b) Let $P(x, y)$ mean that person x plays sport y. Then our statement is $\forall x \exists y\, P(x, y)$.
c) Let $V(x, y)$ mean that person x has visited state y. Then our statement is $\exists x(V(x, \text{Alaska}) \wedge \neg V(x, \text{Hawaii}))$.
d) Let $L(x, y)$ mean that person x has learned programming language y. Then our statement is $\forall x \exists y\, L(x, y)$.
e) Let $T(x, y)$ mean that person x has taken course y, and let $O(y, z)$ mean that course y is offered by department z. Then our statement is $\exists x \exists z \forall y(O(y, z) \to T(x, y))$.
f) Let $G(x, y)$ mean that persons x and y grew up in the same town. Then our statement is $\exists x \exists y(x \neq y \wedge G(x, y) \wedge \forall z(G(x, z) \to (x = y \vee x = z)))$.
g) Let $C(x, y, z)$ mean that persons x and y have chatted with each other in chat group z. Then our statement is $\forall x \exists y \exists z(x \neq y \wedge C(x, y, z))$.

16. We let $P(s, c, m)$ be the statement that student s has class standing c and is majoring in m. The variable s ranges over students in the class, the variable c ranges over the four class standings, and the variable m ranges over all possible majors.
a) The proposition is $\exists s \exists m P(s, \text{junior}, m)$. It is true from the given information.
b) The proposition is $\forall s \exists c P(s, c, \text{computer science})$. This is false, since there are some mathematics majors.
c) The proposition is $\exists s \exists c \exists m\big(P(s, c, m) \wedge (c \neq \text{junior}) \wedge (m \neq \text{mathematics})\big)$. This is true, since there is a sophomore majoring in computer science.
d) The proposition is $\forall s\big(\exists c P(s, c, \text{computer science}) \vee \exists m P(s, \text{sophomore}, m)\big)$. This is false, since there is a freshman mathematics major.
e) The proposition is $\exists m \forall c \exists s P(s, c, m)$. This is false. It cannot be that m is mathematics, since there is no senior mathematics major, and it cannot be that m is computer science, since there is no freshman computer science major. Nor, of course, can m be any other major.

18. a) $\forall f\,(H(f) \to \exists c\, A(c))$, where $A(x)$ means that console x is accessible, and $H(x)$ means that fault condition x is happening
b) $(\forall u \exists m\,(A(m) \wedge S(u, m))) \to \forall u\, R(u)$, where $A(x)$ means that the archive contains message x, $S(x, y)$ means that user x sent message y, and $R(x)$ means that the e-mail address of user x can be retrieved

c) $(\forall b \exists m\, D(m, b)) \leftrightarrow \exists p \neg C(p)$, where $D(x, y)$ means that mechanism x can detect breach y, and $C(x)$ means that process x has been compromised
d) $\forall x \forall y\,(x \neq y \to \exists p \exists q\,(p \neq q \wedge C(p, x, y) \wedge C(q, x, y)))$, where $C(p, x, y)$ means that path p connects endpoint x to endpoint y
e) $\forall x\,((\forall u\, K(x, u)) \leftrightarrow x = \text{SysAdm})$, where $K(x, y)$ means that person x knows the password of user y

20. a) $\forall x \forall y((x < 0) \wedge (y < 0) \to (xy > 0))$ b) $\forall x \forall y((x > 0) \wedge (y > 0) \to ((x + y)/2 > 0))$
c) What does "necessarily" mean in this context? The best explanation is to assert that a certain universal conditional statement is not true. So we have $\neg\forall x \forall y((x < 0) \wedge (y < 0) \to (x - y < 0))$. Note that we do not want to put the negation symbol inside (it is not true that the difference of two negative integers is never negative), nor do we want to negate just the conclusion (it is not true that the sum is always nonnegative). We could rewrite our solution by passing the negation inside, obtaining $\exists x \exists y((x < 0) \wedge (y < 0) \wedge (x - y \geq 0))$.
d) $\forall x \forall y\,(|x + y| \leq |x| + |y|)$

22. $\exists x \forall a \forall b \forall c\,((x > 0) \wedge x \neq a^2 + b^2 + c^2)$, where the domain of discourse consists of all integers

24. a) There exists an additive identity for the real numbers—a number that when added to every number does not change its value.
b) A nonnegative number minus a negative number is positive.
c) The difference of two nonpositive numbers is not necessarily nonpositive.
d) The product of two numbers is nonzero if and only if both factors are nonzero.

26. a) This is false, since $1 + 1 \neq 1 - 1$. b) This is true, since $2 + 0 = 2 - 0$.
c) This is false, since there are many values of y for which $1 + y \neq 1 - y$.
d) This is false, since the equation $x + 2 = x - 2$ has no solution.
e) This is true, since we can take $x = y = 0$. f) This is true, since we can take $y = 0$ for each x.
g) This is true, since we can take $y = 0$. h) This is false, since part (d) was false.
i) This is certainly false.

28. a) true (let $y = x^2$) b) false (no such y exists if x is negative) c) true (let $x = 0$)
d) false (the commutative law for addition always holds) e) true (let $y = 1/x$)
f) false (the reciprocal of y depends on y—there is not one x that works for all y) g) true (let $y = 1 - x$)
h) false (this system of equations is inconsistent)
i) false (this system has only one solution; if $x = 0$, for example, then no y satisfies $y = 2 \wedge -y = 1$)
j) true (let $z = (x + y)/2$)

30. We need to use the transformations shown in Table 2 of Section 1.3, replacing $\neg\forall$ by $\exists\neg$, and replacing $\neg\exists$ by $\forall\neg$. In other words, we push all the negation symbols inside the quantifiers, changing the sense of the quantifiers as we do so, because of the equivalences in Table 2 of Section 1.3. In addition, we need to use De Morgan's laws (Section 1.2) to change the negation of a conjunction to the disjunction of the negations and to change the negation of a disjunction to the conjunction of the negations. We also use the fact that $\neg\neg p \equiv p$.
a) $\forall y \forall x \neg P(x, y)$ b) $\exists x \forall y \neg P(x, y)$ c) $\forall y(\neg Q(y) \vee \exists x\, R(x, y))$
d) $\forall y(\forall x \neg R(x, y) \wedge \exists x \neg S(x, y))$ e) $\forall y(\exists x \forall z \neg T(x, y, z) \wedge \forall x \exists z \neg U(x, y, z))$

32. As we push the negation symbol toward the inside, each quantifier it passes must change its type. For logical connectives we either use De Morgan's laws or recall that $\neg(p \to q) \equiv p \wedge \neg q$ (Table 7 in Section 1.2) and that $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$ (Exercise 21 in Section 1.2).

a)
$$\begin{aligned}\neg\exists z\forall y\forall x\, T(x,y,z) &\equiv \forall z\neg\forall y\forall x\, T(x,y,z)\\ &\equiv \forall z\exists y\neg\forall x\, T(x,y,z)\\ &\equiv \forall z\exists y\exists x\,\neg T(x,y,z)\end{aligned}$$

b)
$$\begin{aligned}\neg(\exists x\exists y\, P(x,y) \land \forall x\forall y\, Q(x,y)) &\equiv \neg\exists x\exists y\, P(x,y) \lor \neg\forall x\forall y\, Q(x,y)\\ &\equiv \forall x\neg\exists y\, P(x,y) \lor \exists x\neg\forall y\, Q(x,y)\\ &\equiv \forall x\forall y\,\neg P(x,y) \lor \exists x\exists y\,\neg Q(x,y)\end{aligned}$$

c)
$$\begin{aligned}\neg\exists x\exists y(Q(x,y) \leftrightarrow Q(y,x)) &\equiv \forall x\neg\exists y(Q(x,y) \leftrightarrow Q(y,x))\\ &\equiv \forall x\forall y\neg(Q(x,y) \leftrightarrow Q(y,x))\\ &\equiv \forall x\forall y(\neg Q(x,y) \leftrightarrow Q(y,x))\end{aligned}$$

d)
$$\begin{aligned}\neg\forall y\exists x\exists z\,(T(x,y,z) \lor Q(x,y)) &\equiv \exists y\neg\exists x\exists z\,(T(x,y,z) \lor Q(x,y))\\ &\equiv \exists y\forall x\neg\exists z\,(T(x,y,z) \lor Q(x,y))\\ &\equiv \exists y\forall x\forall z\,\neg(T(x,y,z) \lor Q(x,y))\\ &\equiv \exists y\forall x\forall z\,(\neg T(x,y,z) \land \neg Q(x,y))\end{aligned}$$

34. The logical expression is asserting that the domain consists of at most two members. (It is saying that whenever you have two unequal objects, any object has to be one of those two. Note that this is vacuously true for domains with one element.) Therefore any domain having one or two members will make it true (such as the female members of the United States Supreme Court in 2005), and any domain with more than two members will make it false (such as all members of the United States Supreme Court in 2005).

36. In each case we need to specify some predicates and identify the domain of discourse.
a) Let $L(x,y)$ mean that person x has lost y dollars playing the lottery. The original statement is then $\neg\exists x\exists y(y > 1000 \land L(x,y))$. Its negation of course is $\exists x\exists y(y > 1000 \land L(x,y))$; someone has lost more than $1000 playing the lottery.
b) Let $C(x,y)$ mean that person x has chatted with person y. The given statement is $\exists x\exists y(y \neq x \land \forall z(z \neq x \to (z = y \leftrightarrow C(x,z))))$. The negation is therefore $\forall x\forall y(y \neq x \to \exists z(z \neq x \land \neg(z = y \leftrightarrow C(x,z))))$. In English, everybody in this class has either chatted with no one else or has chatted with two or more others.
c) Let $E(x,y)$ mean that person x has sent e-mail to person y. The given statement is $\neg\exists x\exists y\exists z(y \neq z \land x \neq y \land x \neq z \land \forall w(w \neq x \to (E(x,w) \leftrightarrow (w = y \lor w = z))))$. The negation is obviously $\exists x\exists y\exists z(y \neq z \land x \neq y \land x \neq z \land \forall w(w \neq x \to (E(x,w) \leftrightarrow (w = y \lor w = z))))$. In English, some student in this class has sent e-mail to exactly two other students in this class.
d) Let $S(x,y)$ mean that student x has solved exercise y. The statement is $\exists x\forall y\, S(x,y)$. The negation is $\forall x\exists y\neg S(x,y)$. In English, for every student in this class, there is some exercise that he or she has not solved. (One could also interpret the given statement as asserting that for every exercise, there exists a student—perhaps a different one for each exercise—who has solved it. In that case the order of the quantifiers would be reversed. Word order in English sometimes makes for a little ambiguity.)
e) Let $S(x,y)$ mean that student x has solved exercise y, and let $B(y,z)$ mean that exercise y is in section z of the book. The statement is $\neg\exists x\forall z\exists y(B(y,z)\land S(x,y))$. The negation is of course $\exists x\forall z\exists y(B(y,z)\land S(x,y))$. In English, some student has solved at least one exercise in every section of this book.

38. a) In English, the negation is "Some student in this class does not like mathematics." With the obvious propositional function, this is $\exists x\neg L(x)$.
b) In English, the negation is "Every student in this class has seen a computer." With the obvious propositional function, this is $\forall x S(x)$.

c) In English, the negation is "For every student in this class, there is a mathematics course that this student has not taken." With the obvious propositional function, this is $\forall x\exists c\neg T(x,c)$.
d) As in Exercise 15f, let $P(z,y)$ be "Room z is in building y," and let $Q(x,z)$ be "Student x has been in room z." Then the original statement is $\exists x\forall y\exists z(P(z,y) \land Q(x,z))$. To form the negation, we change all the quantifiers and put the negation on the inside, then apply De Morgan's law. The negation is therefore $\forall x\exists y\forall z(\neg P(z,y) \lor \neg Q(x,z))$, which is also equivalent to $\forall x\exists y\forall z(P(z,y) \to \neg Q(x,z))$. In English, this could be read, "For every student there is a building such that for every room in that building, the student has not been in that room."

40. a) There are many counterexamples. If $x = 2$, then there is no y among the integers such that $2 = 1/y$, since the only solution of this equation is $y = 1/2$. Even if we were working in the domain of real numbers, $x = 0$ would provide a counterexample, since $0 = 1/y$ for no real number y.
b) We can rewrite $y^2 - x < 100$ as $y^2 < 100 + x$. Since squares can never be negative, no such y exists if x is, say, -200. This x provides a counterexample.
c) This is not true, since sixth powers are both squares and cubes. Trivial counterexamples would include $x = y = 0$ and $x = y = 1$, but we can also take something like $x = 27$ and $y = 9$, since $27^2 = 3^6 = 9^3$.

42. The distributive law is just the statement that $x(y+z) = xy+xz$ for all real numbers. Therefore the expression we want is $\forall x\forall y\forall z\,(x(y+z) = xy+xz)$, where the quantifiers are assumed to range over (i.e., the domain of discourse is) the real numbers.

44. We want to say that for each triple of coefficients (the a, b, and c in the expression $ax^2 + bx + c$, where we insist that $a \neq 0$ so that this actually is quadratic), there are at most two values of x making that expression equal to 0. The domain here is all real numbers. We write $\forall a\forall b\forall c(a \neq 0 \to \forall x_1\forall x_2\forall x_3(ax_1^2 + bx_1 + c = 0 \land ax_2^2 + bx_2 + c = 0 \land ax_3^2 + bx_3 + c = 0) \to (x_1 = x_2 \lor x_1 = x_3 \lor x_2 = x_3))$.

46. This statement says that there is a number that is less than or equal to all squares.
a) This is false, since no matter how small a positive number x we might choose, if we let $y = \sqrt{x/2}$, then $x = 2y^2$, and it will not be true that $x \leq y^2$.
b) This is true, since we can take $x = -1$, for example.
c) This is true, since we can take $x = -1$, for example.

48. We need to show that each of these propositions implies the other. Suppose that $\forall xP(x) \lor \forall xQ(x)$ is true. We want to show that $\forall x\forall y(P(x) \lor Q(y))$ is true. By our hypothesis, one of two things must be true. Either P is universally true, or Q is universally true. In the first case, $\forall x\forall y(P(x) \lor Q(y))$ is true, since the first expression in the disjunction is true, no matter what x and y are; and in the second case, $\forall x\forall y(P(x) \lor Q(y))$ is also true, since now the second expression in the disjunction is true, no matter what x and y are. Next we need to prove the converse. So suppose that $\forall x\forall y(P(x) \lor Q(y))$ is true. We want to show that $\forall xP(x) \lor \forall xQ(x)$ is true. If $\forall xP(x)$ is true, then we are done. Otherwise, $P(x_0)$ must be false for some x_0 in the domain of discourse. For this x_0, then, the hypothesis tells us that $P(x_0) \lor Q(y)$ is true, no matter what y is. Since $P(x_0)$ is false, it must be the case that $Q(y)$ is true for each y. In other words, $\forall yQ(y)$ is true, or, to change the name of the meaningless quantified variable, $\forall xQ(x)$ is true. This certainly implies that $\forall xP(x) \lor \forall xQ(x)$ is true, as desired.

50. a) By Exercises 45 and 46b in Section 1.3, we can simply bring the existential quantifier outside: $\exists x(P(x) \lor Q(x) \lor A)$.
b) By Exercise 48 of the current section, the expression inside the parentheses is logically equivalent to $\forall x\forall y(P(x) \lor Q(y))$. Applying the negation operation, we obtain $\exists x\exists y\neg(P(x) \lor Q(y))$.

c) First we rewrite this using Table 7 in Section 1.2 as $\exists x Q(x) \vee \neg\exists x P(x)$, which is equivalent to $\exists x Q(x) \vee \forall x \neg P(x)$. To combine the existential and universal statements we use Exercise 49b of the current section, obtaining $\forall x \exists y(\neg P(x) \vee Q(y))$, which is in prenex normal form.

52. We simply want to say that there exists an x such that $P(x)$ holds, and that every y such that $P(y)$ holds must be this same x. Thus we write $\exists x\big(P(x) \wedge \forall y(P(y) \rightarrow y = x)\big)$. Even more compactly, we can write $\exists x \forall y(P(y) \leftrightarrow y = x)$.