c) First we rewrite this using Table 7 in Section 1.2 as $\exists x Q(x) \vee \neg\exists x P(x)$, which is equivalent to $\exists x Q(x) \vee \forall x \neg P(x)$. To combine the existential and universal statements we use Exercise 49b of the current section, obtaining $\forall x \exists y(\neg P(x) \vee Q(y))$, which is in prenex normal form.

52. We simply want to say that there exists an x such that $P(x)$ holds, and that every y such that $P(y)$ holds must be this same x. Thus we write $\exists x\big(P(x) \wedge \forall y(P(y) \to y = x)\big)$. Even more compactly, we can write $\exists x \forall y(P(y) \leftrightarrow y = x)$.

SECTION 1.5 Rules of Inference

2. This is modus tollens. The first statement is $p \to q$, where p is "George does not have eight legs" and q is "George is not an insect." The second statement is $\neg q$. The third is $\neg p$. Modus tollens is valid. We can therefore conclude that the conclusion of the argument (third statement) is true, given that the hypotheses (the first two statements) are true.

4. a) We have taken the conjunction of two propositions and asserted one of them. This is, according to Table 1, simplification.
b) We have taken the disjunction of two propositions and the negation of one of them, and asserted the other. This is, according to Table 1, disjunctive syllogism. See Table 1 for the other parts of this exercise as well.
c) modus ponens d) addition e) hypothetical syllogism

6. Let r be the proposition "It rains," let f be the proposition "It is foggy," let s be the proposition "The sailing race will be held," let l be the proposition "The life saving demonstration will go on," and let t be the proposition "The trophy will be awarded." We are given premises $(\neg r \vee \neg f) \to (s \wedge l)$, $s \to t$, and $\neg t$. We want to conclude r. We set up the proof in two columns, with reasons, as in Example 6. Note that it is valid to replace subexpressions by other expressions logically equivalent to them.

Step	Reason
1. $\neg t$	Hypothesis
2. $s \to t$	Hypothesis
3. $\neg s$	Modus tollens using (1) and (2)
4. $(\neg r \vee \neg f) \to (s \wedge l)$	Hypothesis
5. $(\neg(s \wedge l)) \to \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6. $(\neg s \vee \neg l) \to (r \wedge f)$	De Morgan's law and double negative
7. $\neg s \vee \neg l$	Addition, using (3)
8. $r \wedge f$	Modus ponens using (6) and (7)
9. r	Simplification using (8)

8. First we use universal instantiation to conclude from "For all x, if x is a man, then x is not an island" the special case of interest, "If Manhattan is a man, then Manhattan is not an island." Then we form the contrapositive (using also double negative): "If Manhattan is an island, then Manhattan is not a man." Finally we use modus ponens to conclude that Manhattan is not a man. Alternatively, we could apply modus tollens.

10. a) If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.
b) We really can't conclude anything specific here.
c) By universal instantiation, we conclude from the first conditional statement by modus ponens that dragonflies have six legs, and we conclude by modus tollens that spiders are not insects. We could say using existential generalization that, for example, there exists a non-six-legged creature that eats a six-legged creature, and that there exists a non-insect that eats an insect.
d) We can apply universal instantiation to the conditional statement and conclude that if Homer (respectively, Maggie) is a student, then he (she) has an Internet account. Now modus tollens tells us that Homer is not a student. There are no conclusions to be drawn about Maggie.
e) The first conditional statement is that if x is healthy to eat, then x does not taste good. Universal instantiation and modus ponens therefore tell us that tofu does not taste good. The third sentence says that if you eat x, then x tastes good. Therefore the fourth hypothesis already follows (by modus tollens) from the first three. No conclusions can be drawn about cheeseburgers from these statements.
f) By disjunctive syllogism, the first two hypotheses allow us to conclude that I am hallucinating. Therefore by modus ponens we know that I see elephants running down the road.

12. Applying Exercise 11, we want to show that the conclusion r follows from the five premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, $\neg s$, and q. From q and $q \to (u \wedge t)$ we get $u \wedge t$ by modus ponens. From there we get both u and t by simplification (and the commutative law). From u and $u \to p$ we get p by modus ponens. From p and t we get $p \wedge t$ by conjunction. From that and $(p \wedge t) \to (r \vee s)$ we get $r \vee s$ by modus ponens. From that and $\neg s$ we finally get r by disjunctive syllogism.

14. In each case we set up the proof in two columns, with reasons, as in Example 6.
a) Let $c(x)$ be "x is in this class," let $r(x)$ be "x owns a red convertible," and let $t(x)$ be "x has gotten a speeding ticket." We are given premises $c(\text{Linda})$, $r(\text{Linda})$, $\forall x(r(x) \to t(x))$, and we want to conclude $\exists x(c(x) \wedge t(x))$.

Step	Reason
1. $\forall x(r(x) \to t(x))$	Hypothesis
2. $r(\text{Linda}) \to t(\text{Linda})$	Universal instantiation using (1)
3. $r(\text{Linda})$	Hypothesis
4. $t(\text{Linda})$	Modus ponens using (2) and (3)
5. $c(\text{Linda})$	Hypothesis
6. $c(\text{Linda}) \wedge t(\text{Linda})$	Conjunction using (4) and (5)
7. $\exists x(c(x) \wedge t(x))$	Existential generalization using (6)

b) Let $r(x)$ be "x is one of the five roommates listed," let $d(x)$ be "x has taken a course in discrete mathematics," and let $a(x)$ be "x can take a course in algorithms." We are given premises $\forall x(r(x) \to d(x))$ and $\forall x(d(x) \to a(x))$, and we want to conclude $\forall x(r(x) \to a(x))$. In what follows y represents an arbitrary person.

Step	Reason
1. $\forall x(r(x) \to d(x))$	Hypothesis
2. $r(y) \to d(y)$	Universal instantiation using (1)
3. $\forall x(d(x) \to a(x))$	Hypothesis
4. $d(y) \to a(y)$	Universal instantiation using (3)
5. $r(y) \to a(y)$	Hypothetical syllogism using (2) and (4)
6. $\forall x(r(x) \to a(x))$	Universal generalization using (5)

c) Let $s(x)$ be "x is a movie produced by Sayles," let $c(x)$ be "x is a movie about coal miners," and let

$w(x)$ be "movie x is wonderful." We are given premises $\forall x(s(x) \rightarrow w(x))$ and $\exists x(s(x) \wedge c(x))$, and we want to conclude $\exists x(c(x) \wedge w(x))$. In our proof, y represents an unspecified particular movie.

Step	Reason
1. $\exists x(s(x) \wedge c(x))$	Hypothesis
2. $s(y) \wedge c(y)$	Existential instantiation using (1)
3. $s(y)$	Simplification using (2)
4. $\forall x(s(x) \rightarrow w(x))$	Hypothesis
5. $s(y) \rightarrow w(y)$	Universal instantiation using (4)
6. $w(y)$	Modus ponens using (3) and (5)
7. $c(y)$	Simplification using (2)
8. $w(y) \wedge c(y)$	Conjunction using (6) and (7)
9. $\exists x(c(x) \wedge w(x))$	Existential generalization using (8)

d) Let $c(x)$ be "x is in this class," let $f(x)$ be "x has been to France," and let $l(x)$ be "x has visited the Louvre." We are given premises $\exists x(c(x) \wedge f(x))$, $\forall x(f(x) \rightarrow l(x))$, and we want to conclude $\exists x(c(x) \wedge l(x))$. In our proof, y represents an unspecified particular person.

Step	Reason
1. $\exists x(c(x) \wedge f(x))$	Hypothesis
2. $c(y) \wedge f(y)$	Existential instantiation using (1)
3. $f(y)$	Simplification using (2)
4. $c(y)$	Simplification using (2)
5. $\forall x(f(x) \rightarrow l(x))$	Hypothesis
6. $f(y) \rightarrow l(y)$	Universal instantiation using (5)
7. $l(y)$	Modus ponens using (3) and (6)
8. $c(y) \wedge l(y)$	Conjunction using (4) and (7)
9. $\exists x(c(x) \wedge l(x))$	Existential generalization using (8)

16. **a)** This is correct, using universal instantiation and modus tollens.
b) This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.
c) After applying universal instantiation, it contains the fallacy of affirming the conclusion.
d) This is correct, using universal instantiation and modus ponens.

18. We know that *some* s exists that makes $S(s, \text{Max})$ true, but we cannot conclude that Max is one such s. Therefore this first step is invalid.

20. **a)** This is invalid. It is the fallacy of affirming the conclusion. Letting $a = -2$ provides a counterexample.
b) This is valid; it is modus ponens.

22. We will give an argument establishing the conclusion. We want to show that all hummingbirds are small. Let Tweety be an arbitrary hummingbird. We must show that Tweety is small. The first premise implies that if Tweety is a hummingbird, then Tweety is richly colored. Therefore by (universal) modus ponens we can conclude that Tweety is richly colored. The third premise implies that if Tweety does not live on honey, then Tweety is not richly colored. Therefore by (universal) modus tollens we can now conclude that Tweety does live on honey. Finally, the second premise implies that if Tweety is a large bird, then Tweety does not live on honey. Therefore again by (universal) modus tollens we can now conclude that Tweety is not a large bird, i.e., that Tweety is small, as desired. Notice that we invoke universal generalization as the last step.

24. Steps 3 and 5 are incorrect; simplification applies to conjunctions, not disjunctions.

26. We want to show that the conditional statement $P(a) \rightarrow R(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $P(a)$ is true for a particular a, then $R(a)$ is also true. For such an a, by universal modus ponens from the first premise we have $Q(a)$, and then by universal modus ponens from the second premise we have $R(a)$, as desired.

28. We want to show that the conditional statement $\neg R(a) \rightarrow P(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $\neg R(a)$ is true for a particular a, then $P(a)$ is also true. For such an a, universal modus tollens applied to the second premise gives us $\neg(\neg P(a) \wedge Q(a))$. By rules from propositional logic, this gives us $P(a) \vee \neg Q(a)$. By universal generalization from the first premise, we have $P(a) \vee Q(a)$. Now by resolution we can conclude $P(a) \vee P(a)$, which is logically equivalent to $P(a)$, as desired.

30. Let a be "Allen is a good boy"; let h be "Hillary is a good girl"; let d be "David is happy." Then our assumptions are $\neg a \vee h$ and $a \vee d$. Using resolution gives us $h \vee d$, as desired.

32. We apply resolution to give the tautology $(p \vee \mathbf{F}) \wedge (\neg p \vee \mathbf{F}) \rightarrow (\mathbf{F} \vee \mathbf{F})$. The left-hand side is equivalent to $p \wedge \neg p$, since $p \vee \mathbf{F}$ is equivalent to p, and $\neg p \vee \mathbf{F}$ is equivalent to $\neg p$. The right-hand side is equivalent to $\mathbf{F}$. Since the conditional statement is true, and the conclusion is false, it follows that the hypothesis, $p \wedge \neg p$, is false, as desired.

34. Let us use the following letters to stand for the relevant propositions: d for "logic is difficult"; s for "many students like logic"; and e for "mathematics is easy." Then the assumptions are $d \vee \neg s$ and $e \rightarrow \neg d$. Note that the first of these is equivalent to $s \rightarrow d$, since both forms are false if and only if s is true and d is false. In addition, let us note that the second assumption is equivalent to its contrapositive, $d \rightarrow \neg e$. And finally, by combining these two conditional statements, we see that $s \rightarrow \neg e$ also follows from our assumptions.

a) Here we are asked whether we can conclude that $s \rightarrow \neg e$. As we noted above, the answer is yes, this conclusion is valid.

b) The question concerns $\neg e \rightarrow \neg s$. This is equivalent to its contrapositive, $s \rightarrow e$. That doesn't seem to follow from our assumptions, so let's find a case in which the assumptions hold but this conditional statement does not. This conditional statement fails in the case in which s is true and e is false. If we take d to be true as well, then both of our assumptions are true. Therefore this conclusion is not valid.

c) The issue is $\neg e \vee d$, which is equivalent to the conditional statement $e \rightarrow d$. This does *not* follow from our assumptions. If we take d to be false, e to be true, and s to be false, then this proposition is false but our assumptions are true.

d) The issue is $\neg d \vee \neg e$, which is equivalent to the conditional statement $d \rightarrow \neg e$. We noted above that this validly follows from our assumptions.

e) This sentence says $\neg s \rightarrow (\neg e \vee \neg d)$. The only case in which this is false is when s is false and both e and d are true. But in this case, our assumption $e \rightarrow \neg d$ is also violated. Therefore, in all cases in which the assumptions hold, this statement holds as well, so it *is* a valid conclusion.