Stat 210 - Fall 2009 - HW 8 Solutions and Comments

NOTE: Recall that you are only responsible for knowing the material in Problem 1.

1. **Model building and interpretation**

   (a) The 4th indicator is omitted because we only need three indicators to distinguish 4 groups. The intercept identifies the 4th group. Note that it is helpful but not required to mention the collinearity that would occur if we had included the 4th indicator.

   (b) The slopes for the non-region variables are interpreted as usual (the change in the expected value of Y for a one unit change in X **with all other variables held constant**. The "intercept" is the expected SAT score in the "west" region when all covariates are zero (which is not an especially meaningful quantity). The coefficient of "east" is the difference in expected SAT score between "east" and "west" states with all other variables held constant. Similarly for "midwest" and "south".

   (c) This asks for a comparison of Model I and Model II. Note the test of $H_o : \beta_{east} = \beta_{south} = \beta_{midwest} = 0$ tells us whether there are regional differences in SAT scores after controlling for the values of yrs, rank and expend. We compare the full model (II) to the reduced model (I) using $F = ((31708 - 21568)/3)/(21569/43) = 6.74$. When compared to $F_{3,43}$ this yields $p < .001$ indicating that there are regional differences after controlling for yrs, rank and expend.

   (d) Model III assumes that the relationship of SAT and yrs and the relationship of SAT and rank is the same across regions but allows different regional intercepts and different regional slopes with respect to expenditure. You can (and arguably should) write out the model for each region, as in: West -139.3 - .037 expend + 15.2 yrs + 10.5 rank; East -163.4 + 1.75 expend + 15.2 yrs + 10.5 rank; etc..

   (e) This question leads one to compare Models II and III. $F = ((21569 - 19239)/3)/(19239/40) = 1.61$ with $p = .20$ (compared to $F_{3,40}$ distn). Thus there is no evidence of differences in expenditure slope across the regions.

2. **Two-factor factorial** - SAS program is provided below.

   (a) The plot of means shows that the lines connecting the verbal reinforcement means and the nonverbal reinforcement means cross. This suggests that there is an interaction.

   (b) ANOVA table shows reinforcement, isolation, and their interaction are each significant at the .05 level.

Source	DF	Type I SS	Mean Square	F Value	Pr > F
reinforce	1	196.000000	196.000000	12.42	0.0014
isolation	2	156.222222	78.111111	4.95	0.0139
reinforce*isolation	2	1058.666667	529.333333	33.55	<.0001
Error	30	473.333333	15.777778		
Corrected Total	35	1884.222222			

   (c) The residual plots appear consistent with constant variance. The normal probability plot exhibits a fairly straight line indicating the normal error assumption is met.

   (d) Contrast results:

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
linear in isolation	1	6.0000000	6.0000000	0.38	0.5421
quadr in isolation	1	150.2222222	150.2222222	9.52	0.0043

   i. The quadratic contrast for isolation time is significant but the linear contrast is not.

   ii. There is no contrast required for reinforcement since this factor has only two levels and thus 1 d.f. In other words there is only one contrast (weights (-1,1)) and this is the contrast reported in the usual ANOVA output.

   iii. The main effect shows that reinforcement does work to increase memory retention. The interaction suggests that verbal reinforcement does not work equally well for all isolation times. The interaction is the difference in the reinforcement effect over different isolation times (the effect of verbal reinforcement is small (even negative) at 20 and 40 minutes and big at 60 minutes).

   (e) Summary: Positive verbal reinforcement does tend to enhance children's memory. The effect is especially noticeable when there is a long time before the recall event. In fact there is no difference between the reinformcent and non-reinforcement groups at lower isolation times. The results are relevant for 4th graders in the city. It is not clear that we can expand results to include other cities or even other grades.

```
filename datafile 'h:\HAL\courses\Stat210\paragraphdata.txt';
options linesize=80;
data thedata;
   infile datafile firstobs=2;
```

```
    input reinf $ isol score;
proc gplot;
    plot score*isol;
proc glm order=data;
    class reinf isol;
    model score = reinf isol reinf*isol;
    means reinf isol reinf*isol;
    contrast 'isol.lin' isol -1 0 1;
    contrast 'isol.quad' isol -1 2 -1;
    output out=resids r=res p=yhat;
proc rank normal=blom out=norm data=resids;
  var res;
  ranks nrm;
proc gplot data=norm;
    plot res*yhat res*nrm;
run;
```

3. **Three-factor factorial**

(a)

	d.f.	SS	MS	F	pvalue
* species	2	9900.0	4950.0	50.11	0.0000
* soil	2	16436.1	8218.0	83.19	0.0000
* fungicide	1	1932.0	1932.0	19.56	0.0001
spec*soil	4	658.4	164.6	1.67	0.1792
spec*fung	2	194.0	97.0	0.98	0.3844
* soil*fung	2	1851.1	925.6	9.37	0.0005
* threeway	4	1069.7	267.4	2.71	0.0454
error	36	3556.2	98.8		

*--- significant st the .05 level

(b) i. Means for the levels of each main factor are given below. From the means, we see that sweet clover emerges slowly, clay is problematic, and fungicide works.

Species	:	Alfalfa	76.7	Red clover	82.2	Sweet clover	51.0
Soil Type	:	Silt loam	77.8	Sand	86.2	Clay	45.8
Fungicide	:	No	64.0	Yes	75.9		

ii. The ANOVA table tells us that the Soil*Fungicide interaction is most significant. For interpretation, we should look at the interaction means or plot.

		No	Yes (Fungicide)
	Silt loam	74.7	81.0
(Soil)	Sand	85.7	86.7
	Clay	31.7	60.0

Interpretation: Main effects indicate that fungicide is effective. The interaction shows that the fungicide is very effective in clay, less so in the other soils. The fungicide effect is not the same for all soils.

4. **Unbalanced data**

(a) i. As we remarked in class, Type I SS $\neq$ Type III SS for unbalanced data.

ii. The main effects of 'treatment' and 'species' are significant, as is their interaction. Note that residuals show less variance when Y is small. This is common with proportion data. We can use the transformation $\sin^{-1}\sqrt{Y/100}$ to fix this.

(b) i. Possible contrasts of interest (you were only asked for one):

- Treatment 1 vs Treatment 2 (these two treatments would be expected to be equal)
- Treatment 3 vs Treatments 1 & 2 (effect of UV blocking) with weights (-1, -1, 2) for example.

If you carry out those tests then it turns out that the effect of UV blocking is highly significant but the comparison of the two control groups is not. Note that contrasts can be computed directly from the table of means (you need the two way table though) $SS_{cont} = \frac{(\sum_i \sum_j c_{ij}\bar{Y}_{ij.})^2}{\sum_i \sum_j c_{ij}^2/n_{ij}}$ with appropriate choice of the c_{ij}.

ii. Two possible approaches: We can treat the three enzyme levels as 'High', 'Medium' and 'Low', and then use linear (c=(1,0,-1)) and/or quadratic (c=(-1, 2, -1)) contrasts. It is also possible to define contrast weights that take into account the precise enzyme levels, i.e., weights that correspond to a linear pattern in enzyme levels but we won't discuss that here. Here both the linear and quadratic are significant at the .05 level. In fact, the means are nearly equal for the two lower enzyme levels (which explains why it is not just a linear pattern on the high, medium, low scale).

```
filename datafile 'h:\HAL\courses\Stat210\uvfrogsdata.txt';
options linesize=80;
data thedata;
    infile datafile firstobs=2;
```

```
    input pctfail treat location species;
proc glm order=data;
    class treat species;
    model pctfail = treat species treat*species;
    means reinf isol reinf*isol;
    output out=resids r=res p=yhat;
proc rank normal=blom out=norm data=resids;
  var res;
  ranks nrm;
proc gplot data=norm;
    plot res*yhat res*nrm;
run;
```

5. **Two-factor factorial with blocking**

   (a) Recall that ANOVA assumes the variance of the responses in different treatments (or groups in this case) is the same. This is certainly not true of the raw numbers. It appears that the numbers in the four groups can't be easily compared due to their different magnitudes.

   (b) The s.d.'s of the four groups are much more similar on the log scale. This often occurs when effects are multiplicative. For example if a trip is good then we may catch 10x as many fish and would expect 10x as many in each category. The logarithm reduces these multiplicative block effects to additive effects.

   (c) The F-statistics are: F for spec = 2506.4 with $P < .0001$ (more spec B than spec A); F for sex = 376.0 with $P < .0001$ (more female for each spec); F for interaction = 4.29 with $P = .046$ (significant interaction but not nearly as signif as before logs) The logarithms make the interaction much less significant. The effects of sex and species are still substantial.

   (d) The main effects suggest that more species B fish are caught, and more females are caught. The interaction indicates that the sex difference is different for the two species (more pronounced in species B).

   (e) Blocking appears to be effective. It is highly significant. Recall that even if it is not significant it may still be effective as long as it is reducing the MSE.