Causal inference through potential outcomes framework
- Action (manipulation, treatment, intervention) applied to unit/object (people, class, object)
- Action typically has 2 or more levels
- Outcome - response measured after action
- Potential outcome framework - unit has an outcome under each observation

Example:

Treatment:
- A = take aspirin
- N = no aspirin
- Y = outcome (H=headache, C=cured)

Potential Outcomes:
- Y(A) = outcome on aspirin
- Y(N) = outcome with no aspirin
- Y = observed outcome
- W = treatment indicator = 1 if A; 0 if N

\[ Y = W \times Y(A) + (1 - W) \times Y(N) \]

Everyone starts with a headache and there are 4 possibilities for potential outcomes:

<table>
<thead>
<tr>
<th>Y(A)</th>
<th>Y(N)</th>
<th>Causal Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>Aspirin doesn’t help</td>
</tr>
<tr>
<td>H</td>
<td>C</td>
<td>Aspirin is “harmful”</td>
</tr>
<tr>
<td>C</td>
<td>H</td>
<td>Aspirin helps</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>Aspirin doesn’t help</td>
</tr>
</tbody>
</table>

Unit level causal effect

\[ Y(\text{trt}) - Y(\text{control}) \]
\[ Y(\text{trt}) / Y(\text{control}) \]

- Fundamental Problem – only observe one of the potential outcomes (missing data problem)
  - Need multiple units
  - But even multiple units don’t provide a quick solution:

Consider 2 units \( Y_1(A \ A) \), where person 1 gets treatment A and person 2 also gets treatment A

\[
Y_1(A \ A) \quad Y_1(A \ N) \quad Y_1(N \ A) \quad Y_1(N \ N) \\
Y_2(A \ A) \quad Y_2(A \ N) \quad Y_2(N \ A) \quad Y_2(N \ N)
\]
SUTVA: Stable Unit Treatment Value Assumption

- No interference – potential outcomes for a unit don’t depend on treatment given other units
  
  Example:  
  \[ Y_1(AA) = Y_1(AN) \]  
  \[ Y_1(NA) = Y_1(NN) \]

- Stability / no variation in treatments
  
  - \( Y_1(A) \) is well defined
  
  - No two versions of aspirin, \( Y_1(A) \neq Y_1(A^*) \)

With SUTVA:  

<table>
<thead>
<tr>
<th>Unit 1</th>
<th>Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_1(A) )</td>
<td>( Y_2(A) )</td>
</tr>
<tr>
<td>( Y_1(N) )</td>
<td>( Y_2(N) )</td>
</tr>
</tbody>
</table>

Causal Estimand

Have causal effect for an individual: \( Y(\text{treatment}) - Y(\text{control}) \)

Consider a population of units: \( i = 1, ..., N \)

Causal estimand is a population quantity that summarized unit level causal effects

\[
\tau_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} (Y_i(1) - Y_i(0))
\]

\[
\tau_{\text{med}} = \text{med}(Y_i(1) - Y_i(0))
\]

Assignment Mechanism

How treatments are assigned to units

\( W_i = \) treatment assigned to unit \( i \)

Example: Medical trial with 4 patients

<table>
<thead>
<tr>
<th>Patient</th>
<th>S (surgery)</th>
<th>D (drug)</th>
<th>Causal Effect</th>
<th>( Y(S) - Y(D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td></td>
<td>+6</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td></td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>1</td>
<td></td>
<td>+4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
<td></td>
<td>-1</td>
</tr>
</tbody>
</table>

Avg. trt. Effect = +2

Perfect Doctor

\[
\begin{align*}
W_1 &= S & Y_1 &= Y_1(S) = 7 \\
W_2 &= D & Y_2 &= Y_2(D) = 6 \\
W_3 &= S & Y_3 &= 5 \\
W_4 &= D & Y_4 &= 8
\end{align*}
\]
• With perfect doctor you get the wrong conclusion that drugs work better than surgery
• Clearly $W$ plays a key role in causal inference

**Covariates / Pre-treatment variables**

$X_i = k \times 1$ vector of covariates for unit $i$
Not affected by treatment (assumption – usually attributes of unit determined before treatment is assigned)

Three uses:

1. Can make estimates of causal effects (consider units with some $X_i$)

   Example:
   $X_i = \text{pretest}$
   $Y_i(T) = \text{post - test on treatment } T$
   $Z_i(T) = Y_i(T) - X_i = \text{change - score on } T$

   Individual causal effect:
   
   $$Z_i(T) - Z_i(C) = (Y_i(T) - X_i) - (Y_i(C) - X_i)$$
   $$= Y_i(T) - Y_i(C)$$

2. Causal estimands on subgroups defined by $X_i$

   $$\tau_{\text{avg,}F} = \frac{1}{N_F} \sum_{i=1}^{N} (Y_i(T) - Y_i(C))$$
   $$X_i = F$$

3. In observational studies covariates may help explain treatment assignment/selection and allow better causal estimates.