1 Assessing Unconfoundedness Assumption

- Reading: Chapter 20, Section 12.6, Imbens 2004
- Setting is observational studies with regular assignment mechanism (the unconfounded case)
- Unconfoundedness assumption \( W \perp Y(0), Y(1) | X \) is not directly testable because \( Y(0) \) and \( Y(1) \) are not both observed
- Two approaches to assessing the assumption

1. **use an ‘unaffected outcome’**

   - break covariate vector into two components
     - suppose we take \( X^p = \) pseudo outcome (e.g., lagged value of \( Y \))
     - \( X^r = \) remainder of covariates
   - effect of treatment on \( X^p \) is zero, \( \tau_{X^p} = 0 \) (it is determined pre-treatment assignment)
   - test/estimate \( \tau_{X^p} \) using unconfoundedness assumption on \( X^r \) and see if it is 0
   - if \( \hat{\tau}_{X^p} \approx 0 \) unconfoundedness is plausible
   - if \( \hat{\tau}_{X^p} \neq 0 \) question unconfoundedness

   **The argument for this approach**
   - Goal: \( Y_i(0), Y_i(1) \perp W_i | X_i \)
   - Related condition
     - \( Y_i(0), Y_i(1) \perp X^r_i \)
     - use \( X^p \) as a proxy for \( Y \), note \( X^p_i(0) = X^p_i(1) \) (pretreatment covariates)
   - Note we want to check \( X^p \perp W | X^r \), i.e., check independence and not just check that the average treatment effect = 0
   - to check independence we should examine other estimands for example: \( E(\log X^p(0)) = E(\log X^p(1)); X^p(0) = X^p(1) \) for any subgroup

2. **multiple control groups**

\[
\begin{array}{c|c|c}
W & | & \\
1 & 1 & trt \\
0_{c_1} & ctl_1 & (e.g., not eligible) \\
0_{c_2} & ctl_2 & (e.g., eligible but didn’t participate) \\
\end{array}
\]
Argument
unconfoundedness $Y_i(0)Y_i(1) \perp W_i|X_i$

stronger assumption $Y_i(0)Y_i(1) \perp G_i|X_i, G_i$ group $\in \{1, 0_{c_1}, 0_{c_2}\}$

‘s stronger assumption’ implies $Y_i^{obs} \perp G_i|X_i, G_i \in \{0_{c_1}, 0_{c_2}\}$ which is testable

– causal analysis (matching, propensity score) to compare two control groups given $X_i$
– same comments about practicality of doing this as discussed earlier (for approach 1)

2 Evidence regarding unconfoundedness in practice / Evaluation research

Lalonde (1986)
– randomized experiment (National Supported Work program)
– threw out randomized controls, used data from CPS and PSID as observation controls,
tried various unconfounded strategies for inference
– Lalonde found poor result
– Dehejia & Wahbe re-examined and found better results

Shadish (2008)
– randomized experiment; students randomized into two groups ((a) and (b))

(a) participate in randomized study: math tutoring, vocabulary tutoring
(b) participate in observational study: math tutoring, vocabulary tutoring

$Y_v =$ vocab score, $Y_m =$ math score
can get unbiased treatment effect estimates from randomized study
ask whether ”unconfounded analysis” can reproduce randomized estimates
– conclusion is that methods work well

3 Irregular Assignment Mechanisms

Part IV of Imbens & Rubin, we only have Chapter 24
Gelman & Hill – Chapter 10 (10.4 - 10.7)
Little, Long, Lin – Biometrics 2009
Angrist & Pischke – regression discontinuity + instrumental variables

For irregular assignment mechanisms conditioning on covariates is not enough to obtain causal inferences. We usually require additional assumptions. Four topics come to mind; we discuss the first two in some detail and provide a pointer to learn more about the others.

The four topics:
– Bounds
– Instrumental Variables
Regression Discontinuity Design (Ch 10.6 in G&H)
Difference in differences (Ch 10.7 in G&H)

**Bounds**

\[
\tau = E[Y(1) - Y(0)]
\]

\[
= E[Y(1)|W = 1]Pr(W = 1) + E[Y(1)|W = 0]Pr(W = 0)
- E[Y(0)|W = 1]Pr(W = 1) - E[Y(0)|W = 0]Pr(W = 0)
\]

Don’t have any observed information regarding \(E[Y(1)|W = 0]\) and \(E[Y(0)|W = 1]\)
sometimes we can bound these quantities

if \(Y \in [0, 1]\), then \(0 \leq E[Y(1)|W = 0] \leq 1, 0 \leq E[Y(0)|W = 1] \leq 1\)

\[
\tau \leq E[Y(1)|W = 1]Pr(W = 1) + Pr(W = 0) - E[Y(0)|W = 0]Pr(W = 0)
\]

\[
\geq E[Y(1)|W = 1]Pr(W = 1) - Pr(W = 1) - E[Y(0)|W = 0]Pr(W = 0)
\]

these inequalities are of limited use because interval width is 1 and the interval always contains 0

sometimes we can do better than this
eg., may have reason to believe \(E[Y(1)|W = 1] \geq E[Y(1)|W = 0]\)