Irregular Assignment Mechanism

(Continued from lecture 3/3)

Example from last lecture: Trial – effect of vitamin A on infant mortality

Assignment Z	Trt received W	Outcome Y	# children	
0	0	0	74	
0	0	1	11514	.9936
1	0	0	34	
1	0	1	2385	.9859
1	1	0	12	
1	1	1	9663	.9988

- $ITT_{z \text{ on } Y} = \frac{12048}{12094} - \frac{11514}{11588} = 0.0026$ (compare Z = 1 to Z = 0, on Y)
- $ITT_{z \text{ on } w} = \frac{9675}{12094} - \frac{0}{11588} = 0.80$ (compare Z =1 to Z = 0, on W)

- Per protocol =.9988 - .9936 = 0.0052 (compare w = z = 1 to w = z = 0, on Y)
- As treated =.9988 – (13899/14007) = 0.0065 (compare w = 1 to w = 0)

If we are interested in the effect of W on Y, then there is a problem in that assignment of W is not randomized or even necessarily un-confounded.

The specific concern is that we don't know about the Z = 1, W = 0, and Z = 0, W = 0 groups, we don't know whether these two groups are the same or not.

Since we have no covariate to condition on, the best thing we could do is to control for compliance.

Approach: 2 steps

1. Define "compliance" in a way that we can condition on it
2. We estimate treatment effect of W on a subgroup with some assumptions.

Step 1, define compliance

Four subclasses	**Z = 0**	**Z = 1**	**Notes**
Complier	W = 0	W = 1	
Always taker	W = 1	W = 1	*Empty set in our example*
Never taker	W = 0	W = 0	*Also called non-compliers in our example*
Defier	W = 1	W = 0	*Generally assumed empty*

Compliance

- Property of the unit
- Defined for both Z's
- Not observed for all units

Step 2, now we can talk about causal inference for W on Y in compliance strata.

W is un-confounded given compliance strata, as shown:

$P(W_i^{obs} = 1 | y(0,0), y(1,0), y(0,1), y(1,1), C_i) = P(W_i^{obs} = 1 | C_i)$

If C_i = non-complier/never taker, LHS = RHS = 0

If C_i = complier, $W_i = Z_i$ and LHS = RHS by randomization of Z

If we could condition on C, then earlier work (matching/stratification) solves the problem.

Don't observe C for everyone. But can learn about causal effect on the subgroup of compliers (with some assumptions)

From last time,

$ITT_{z\ on\ y} = ITT_{z\ on\ y,c} \times \pi_c + ITT_{z\ on\ y,n} \times \pi_n$

Add exclusion restrictions:

$y_i(0,0) = y_i(1,0)$, for $c_i = n$, which means that $ITT_{z\ on\ y,n} = 0$

Then we obtain:

$ITT_{z\ on\ y,c}$ = treatment effect of W on Y for compliers (Z=W, for C_i=c)

$$= \frac{ITT_{z\ on\ y}}{\pi_c} = \frac{ITT_{z\ on\ y}}{ITT_{z\ on\ w}} = \frac{0.0026}{0.8} = 0.0032$$

= Compiler Average Causal Effect (CACE) or

Local Average Treatment Effect(LATE)

Variance for estimate: could be estimated using delta method and the fact that the estimate is derived as a function of sample proportions.

Instrumental Variable Approach

Previous example is an instance of the instrumental variable approach. We now describe the approach in more general terms.

Instrument: variable that randomly induces variation in the treatment of interest

(e.g., Z = randomization to treatment group is an instrument in the Vitamin A study)

Key assumptions for IV approach (Gelman & Hill, Chap 10.5, 10.6)

- unconfoundedness of instrument
  - trivially true in example because Z is randomized;
  - it may require use of covariates
- non-zero association between instrument and treatment
  - this can be checked using correlation of Z and W.

    For $ITT_{y,c} = \dfrac{ITT_y}{ITT_w} \longrightarrow$ the effect of randomization on treatment

    If $ITT_w = 0$, it doesn't help
- monotonicity
  - no defiers
- exclusion restriction
  - no direct effect of instrument on the outcome

For future reference we now note that the CACE can be expressed as the ratio of "regression estimates"

CACE = $ITT_{y,c} = \dfrac{ITT_y}{ITT_w}$

$$\longrightarrow \quad \hat{ITT}_{y,c} = \frac{\hat{ITT}_y}{\hat{ITT}_w} = \frac{\bar{y}_{z=1} - \bar{y}_{z=0}}{\bar{w}_{z=1} - \bar{w}_{z=0}} = \frac{\hat{\beta}_{reg\ y\ on\ z}}{\beta_{reg\ w\ on\ z}}$$

Regression view of the IV approach

Outcome Y
Instrument Z } all can be continuous
Treatment T (T is W we used in lecture before)

- We build two regression models: Y on T and T on Z

$Y = \beta_0 + \beta_1 T + \varepsilon$

T may be correlated with error (which is why we can't estimate effect of T using this regression)

$T = \gamma_0 + \gamma_1 Z + \nu$

- Four assumptions of IV in this regression setup:

Non-zero association: $\gamma_1 \neq 0$

Monotonicity : linearity of relationship of T on Z

Unconfoundedness : Z is independent of ν

(assuming no covariates X, but we can also accommodate X if present)

Exclusion restriction: Z is independent of Y_i and ε, given T_i

IV estimation approach

Regress T on Z $\Rightarrow \hat{\gamma}_0, \hat{\gamma}_1 \Rightarrow \hat{T}$

Regress Y on $\hat{T} \Rightarrow \hat{\beta}_1$

Standard error requires some adjustments

- Where does exclusion restriction come from? Start with more general regression

(no ER)

$Y = \beta_0 + \beta_1 T + \beta_2 Z + \varepsilon$

$T = \gamma_0 + \gamma_1 Z + \nu$

$\longrightarrow$ $Y = (\beta_0 + \beta_1\gamma_0) + (\beta_2 + \beta_1\gamma_1)Z + \varepsilon$, let $\beta_2 + \beta_1 * \gamma_1 = \delta_1$

We want to obtain $\beta_1 = \dfrac{\delta_1 - \beta_2}{\gamma_1}$

Can estimate δ_1 and γ_1 , because Z is randomized

But no unbiased estimate for β_2, since T maybe correlated with ε, so we need exclusion restriction to let $\beta_2 = 0$.