1. Order the following functions by their asymptotic behavior. Circle the groups of functions that are Big-$\Theta$ to each other.

\[
\begin{align*}
a(x) &= x & b(x) &= \sum_{i=0}^{x} 2^i & c(x) &= 2^x \\
d(x) &= 3^x & e(x) &= x \log x & f(x) &= \sum_{i=1}^{x} i \\
g(x) &= x^{1.00001} & h(x) &= x^2 & i(x) &= \sqrt{x} \\
j(x) &= \log_2 3^x & k(x) &= x^{1/\log x} & \ell(x) &= \log x 2^x
\end{align*}
\]

2. For each sorting algorithm below what order of the numbers from 1 to 8 minimizes the number of comparisons performed? What order maximizes the number of comparisons performed? How many comparisons are performed on each list?

- Insertion sort
- Merge sort

3. For each of the following problems decide whether or not the problem can be solved faster on sorted rather than unsorted data (i.e. does sorting the data speed up the time to solve the problem):

- Searching for a specific value in the array
- Finding the mean of an array of ints
- Finding the median of an array of ints
- Finding the mode of an array of ints

4. Recall an inversion is a pair of elements whose current order is opposite of sorted. As discussed in class, the number of inversions in an array can be counted using insertion sort in $O(n^2)$ time. Modify merge sort to count the number of inversions in an array in $O(n \log n)$ time.