CSP

1) Consider the following binary constraint network: There are 4 variables: $X_1$, $X_2$, $X_3$, $X_4$ with the domains: $D_1=\{1,2,3,4\}$, $D_2=\{3,4,5,8,9\}$, $D_3=\{2,3,5,6,7,9\}$, $D_4=\{3,5,7,8,9\}$. The constraints are $X_1 \geq X_2$, $X_2 > X_3$ or $X_3 - X_2 = 2$, $X_3 \neq X_4$.

   a. Draw the constraint graph.

   b. Is the network arc-consistent? If not, compute the arc-consistent network.
      (show the whole process of enforcing arc-consistency and not just the final network)

   c. Is the network consistent? If yes, give a solution.

2) Consider the 8 squares positioned as follows:

   ![8 squares diagram]

   The task is to label the boxes above with the numbers 1-8 such that the labels of any pair of adjacent squares (i.e. horizontal, vertical or diagonal) differ by at least 2 (i.e. 2 or more).

   a. Write all constraints and draw the constraint graph.

   b. Is the network arc-consistent? If not, compute the arc-consistent network.
      (show the whole process of enforcing arc-consistency and not just the final arc-consistent network)

   c. Is the network consistent? If yes, give a solution.
**Variables** \( X_i \) can take values \{0,1,2\}.

* Constraints \( C_1,...,C_5 \) enforce the fact that the variables in their arguments must all have different values.

* Unary constraints \( C_6,C_7 \) enforce:
  - \( C_6 \): \( X_1 = 0 \);
  - \( C_7 \): \( X_4 = \{1,2\} \)

In the following you must always use the “minimum remaining values values heuristic” (MRV) to choose a variable. Use the “degree heuristic” (DH) as a tie-breaker. To assign a value you must use the least constraining value heuristic (LCV).

1. [2pts] Write out the domains of all variables.
   - Use the unary constraints to simplify the domains of \( X_1,X_4 \).
2. [2pts] Use MRV & DH to choose the first variable and assign it a value.
3. [4pts] Use forward checking to simplify the domains of the “neighbors” of that variable.
   - (neighbors are all variables that share some constraint with that variable.)
4. [4pts] Simplify the constraint graph by eliminating the first assigned variable from it.
   - Draw the graph as a “standard constraint graph” (without the constraint boxes) where edges represent constraints between pairs of nodes.
5. [2pts] Use MRV & DH to choose the next variable and assign it a value.
6. [4pts] Make the entire graph arc-consistent. Provide a solution to the CSP.
7. [4pts] Express the time complexity of arc-consistency in terms of \( d \) (number of states) and \( n \) (number of nodes) for general graphs. Is arc-consistency exponential or polynomial in \( d,n \)?
8. [2pts] Same as 7 but now for a tree-structured constraint graph.