CompSci 171: Intro AI

Homework 4

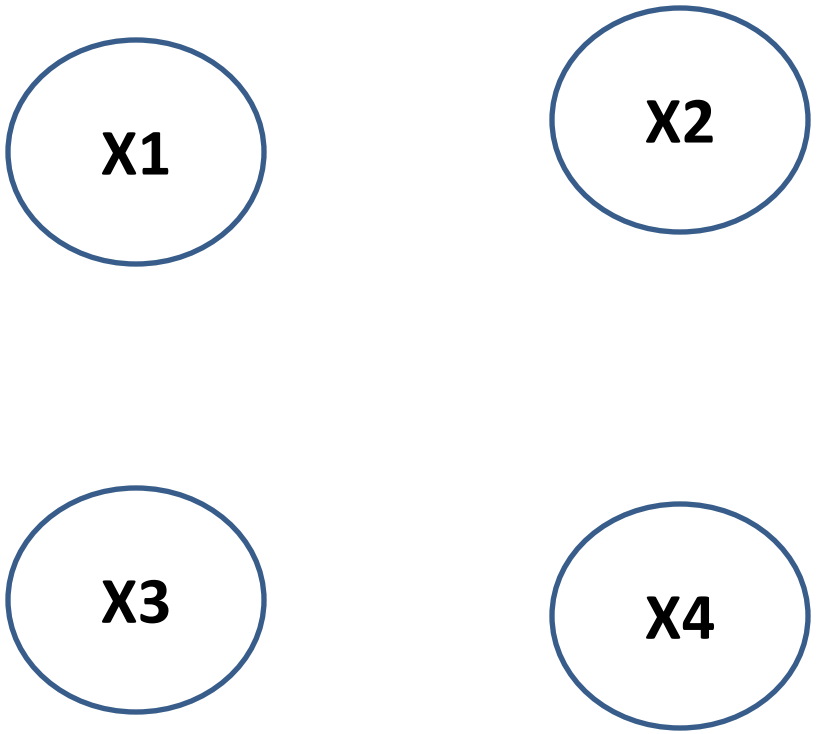
Constraint Satisfaction Problems

1.a – Draw the constraint graph

- 4 variables: X1, X2, X3, X4
- Domains:
  - D1 = {1, 2, 3, 4}
  - D2 = {3, 4, 5, 8, 9}
  - D3 = {2, 3, 5, 6, 7, 9}
  - D4 = {3, 5, 7, 8, 9}
- Constraints:
    - X1 >=X2
    - X2 > X3 or (X3 – X2) = 2
    - X3 <> X4

1.a – Draw the constraint graph

4 variables: X1, X2, X3, X4 => **4 nodes**



1.a – Draw the constraint graph

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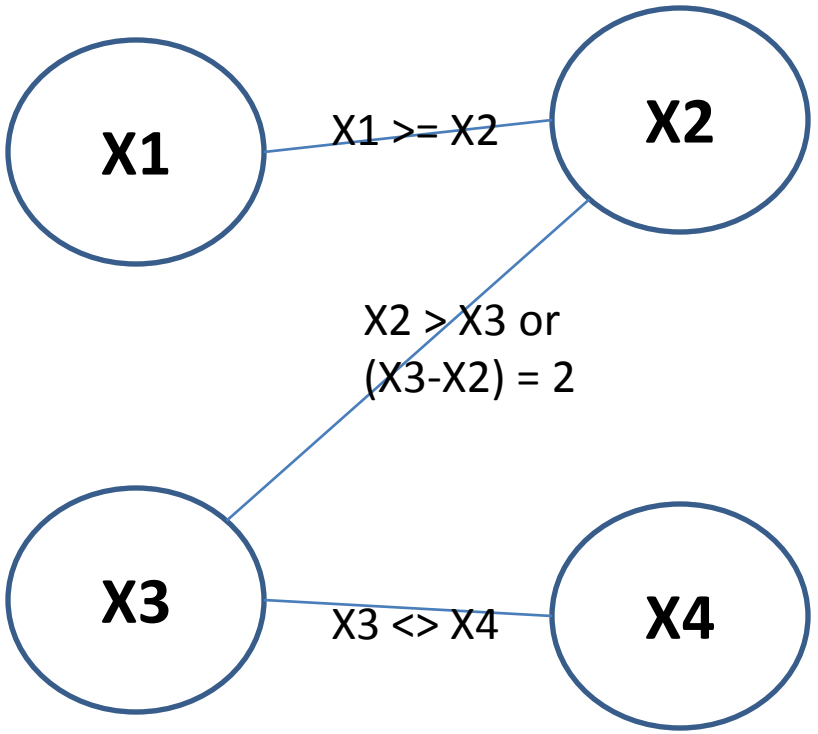
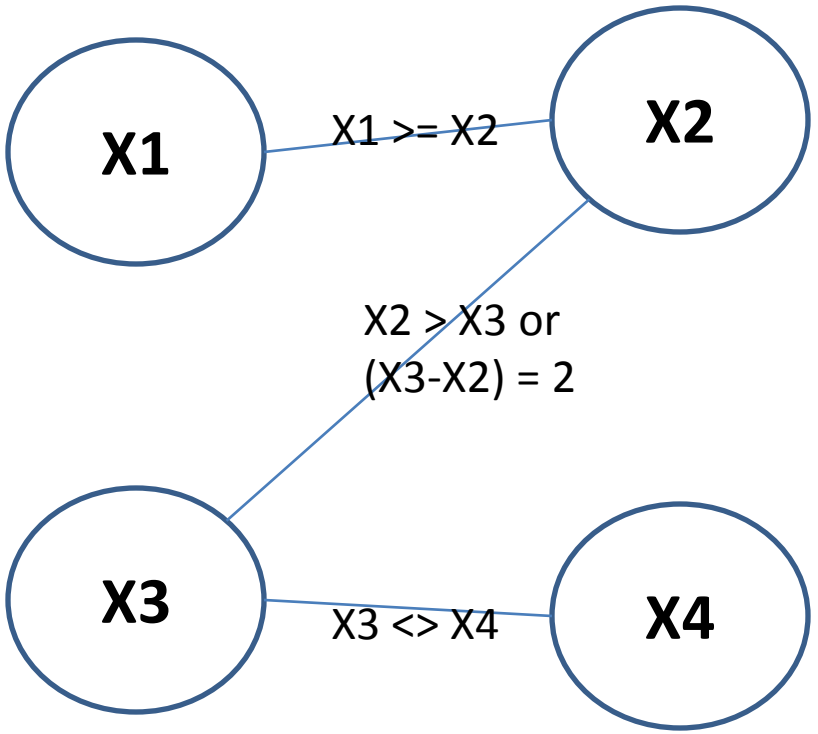


1.b – Is the network arc-consistent?

4 variables: X1, X2, X3, X4 => **4 nodes**

Constraints:

- X1 >=X2
- X2 > X3 or
- (X3 – X2) = 2
- X3 <> X4



1.b –Is the network arc-consistent?

Arc X -> Y is consistent iff for *every* value x of X there is *some* allowed y of Y

$$X1 = 1 < D2 = \{3,4,5,8,9\}$$

Not arc-consistent!

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: (x3,x4), (x2,x3), (x1,x2), (x2,x1), (x3,x2), (x4,x3)

- Check (x3,x4)
  - D3 = {2, 3, 5, 6, 7, 9}
  - D4 = {3, 5, 7, 8, 9}

  Constraint: X3 <> X4

  No value is removed from D3

Domains:
– D1 = {1, 2, 3, 4}
– D2 = {3, 4, 5, 8, 9}
– D3 = {2, 3, 5, 6, 7, 9}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

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- Check (x2,x3)
  - D2 = {3, 4, 5, 8, 9}
  - D3 = {2, 3, 5, 6, 7, 9}

  Constraint: X2 > X3 or (X3 –X2) = 2

  No value is removed from D2

Domains:
– D1 = {1, 2, 3, 4}
– D2 = {3, 4, 5, 8, 9}
– D3 = {2, 3, 5, 6, 7, 9}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: (x1,x2), (x2,x1), (x3,x2), (x4,x3)

- Check (x1,x2)
  - D1 = {1, 2, 3, 4}
  - D2 = {3, 4, 5, 8, 9}

  Constraint: X1 >=X2

  Remove {1,2} from D1

Domains:
– D1 = {1, 2, 3, 4}
– D2 = {3, 4, 5, 8, 9}
– D3 = {2, 3, 5, 6, 7, 9}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: (x2,x1), (x3,x2), (x4,x3)

- Check (x2,x1)
  - D2 = {3, 4, 5, 8, 9}
  - D1 = {3, 4}

  Constraint: X1 >= X2

  Remove {5,8,9} from D2

Domains:
– D1 = {3, 4}
– D2 = {3, 4, 5, 8, 9}
– D3 = {2, 3, 5, 6, 7, 9}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: (x3,x2), (x4,x3)

- Check (x3,x2)
  - D3 = {2, 3, 5, 6, 7, 9}
  - D2 = {3, 4}

  Constraint: X2 > X3 or (X3 –X2) = 2

  Remove {7,9} from D3

Domains:
– D1 = {3, 4}
– D2 = {3, 4}
– D3 = {2, 3, 5, 6, 7, 9}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: (x4,x3)

- Check (x4,x3)
  - D4 = {3, 5, 7, 8, 9}
  - D3 = {2, 3, 5, 6}

  Constraint: X3 <> X4

  No value is removed from D4

Domains:
– D1 = {3, 4}
– D2 = {3, 4}
– D3 = {2, 3, 5, 6}
– D4 = {3, 5, 7, 8, 9}

1.b –Is the network arc-consistent?

Run AC-3 Algorithm

queue: Null

- Arc consistent

Domains:
– D1 = {3, 4}
– D2 = {3, 4}
– D3 = {2, 3, 5, 6}
– D4 = {3, 5, 7, 8, 9}

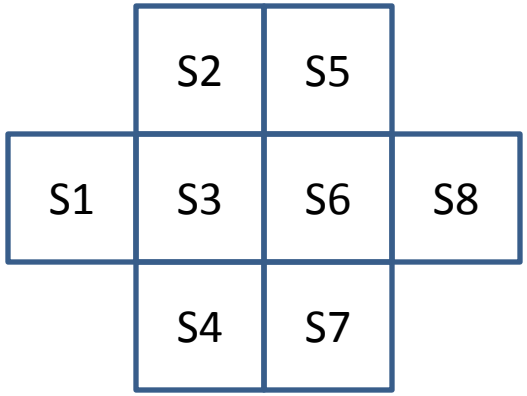
1.c – Give a solution

- 4 variables: X1, X2, X3, X4
- Constraints:
  - X1 >=X2
  - X2 > X3 or (X3 – X2) = 2
  - X3 <> X4
- Network is arc-consistent for domains:
  - D1’ = {3, 4}
  - D2’ = {3, 4}
  - D3’ = {2, 3, 5, 6}
  - D4’ = {3, 5, 7, 8, 9}
- One of possible solutions is:

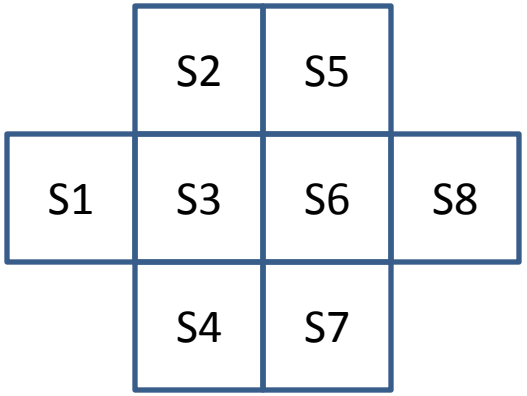
X1 = 3; X2 = 3; X3 = 2; X4 = 3

2. 8 boxes problem

Label the boxes with numbers 1 - 8, such that the labels of any pairs of adjacent squares (horizontal, vertical, diagonal), differ by at least 2



2.a (V2) write all constraints



8 variables: S1, S2, S3, S4, S5, S6, S7, S8

Domains Di = {1, 2, 3, 4, 5, 6, 7, 8}, i=(1,8)

Constraints:

S1: |S1 – S2| >= 2 and |S1 – S3| >= 2 and |S1 – S4| >= 2;

S2: |S1 – S2| >= 2 and |S2 – S3| >= 2 and |S2 – S5| >= 2 and |S2 – S6| >= 2;

S3: |S1 - S3| >= 2 and |S2 – S3| >= 2 and |S3 – S4| >= 2 and |S3 – S5| >= 2 and |S3 – S6| >= 2 and |S3 – S7| >= 2;

S4: |S1 – S4| >= 2 and |S3 – S4| >= 2 and |S4 – S6| >= 2 and |S4 – S7| >= 2;

S5: |S2 – S5| >= 2 and |S3 – S5| >= 2 and |S5 – S6| >= 2;

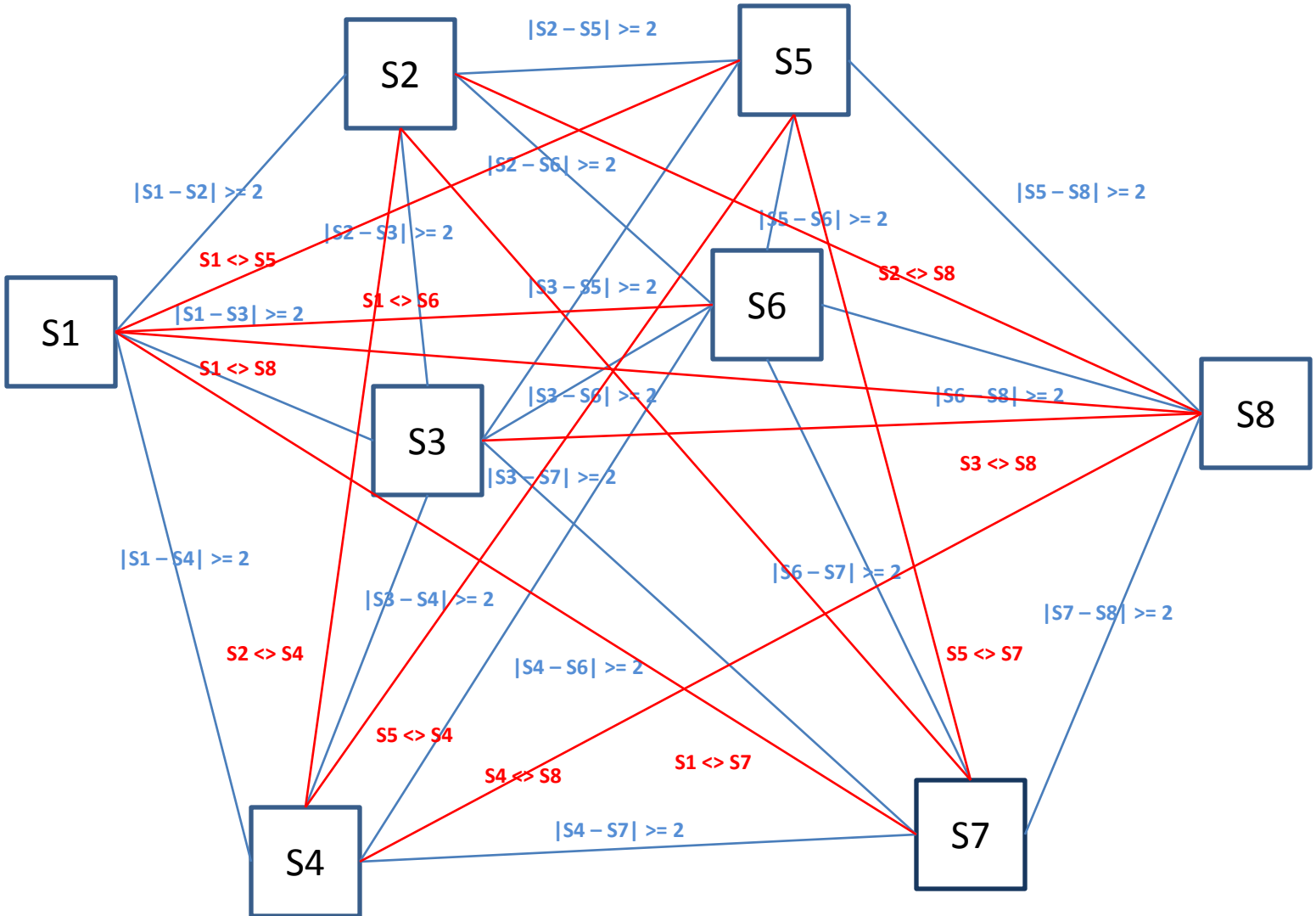
S6: |S2 – S6| >= 2 and |S3 – S6| >= 2 and |S4 – S6| >= 2 and |S5 – S6| >= 2 and |S6 – S7| >= 2 and |S6 – S8| >= 2;

S7: |S3 – S7| >= 2 and |S4 – S7| >= 2 and |S6 – S7| >= 2 and |S7 – S8| >= 2;

S8: |S5 – S8| >= 2 and |S6 – S8| >= 2 and |S7 – S8| >= 2

S1 <> S2 <> S3 <> S4 <> S5 <> S6 <> S7 <> S8

2.a Draw constrain graph



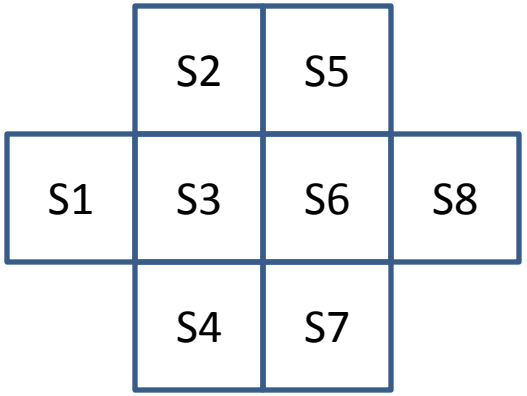
2.a Draw constrain graph

Blue lines:

|S1 – S2| >= 2;
|S1 – S3| >= 2;
|S1 – S4| >= 2;
|S2 – S3| >= 2;
|S2 – S5| >= 2;
|S2 – S6| >= 2;
|S3 – S4| >= 2;
|S3 – S5| >= 2;
|S3 – S6| >= 2;
|S3 – S7| >= 2;
|S4 – S6| >= 2;
|S4 – S7| >= 2;
|S5 – S6| >= 2;
|S6 – S7| >= 2;
|S6 – S8| >= 2;
|S7 – S8| >= 2.

Red lines:

S1 <> S5;
S1 <> S6;
S1 <> S7;
S1 <> S8;
S2 <> S4;
S2 <> S7;
S2 <> S8;
S3 <> S8;
S4 <> S5;
S4 <> S8;
S5 <> S7.



2.b Is the network arc-consistent?

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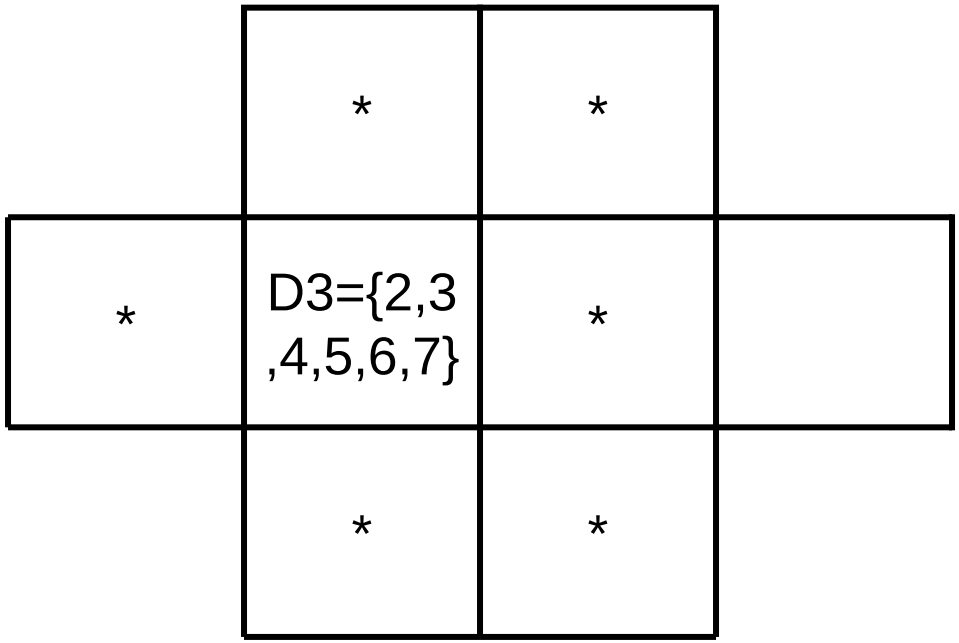
Yes

For any arc (Si, Sj), for any value in the domain of Di, we can find some value in Dj such that |Si-Sj|>=2 (and consequently Si<>Sj).

E.g., for any value in Di={1, 2, 3, 4, 5, 6, 7, 8}, either Di-2 or Di+2 is in Dj={1, 2, 3, 4, 5, 6, 7, 8}.

2.c Is the network consistent?

D3 = {1,2,3,4,5,6,7,8}



D* = {1,2,3,4,5,6,7,8} – {S3-1, S3, S3+1}

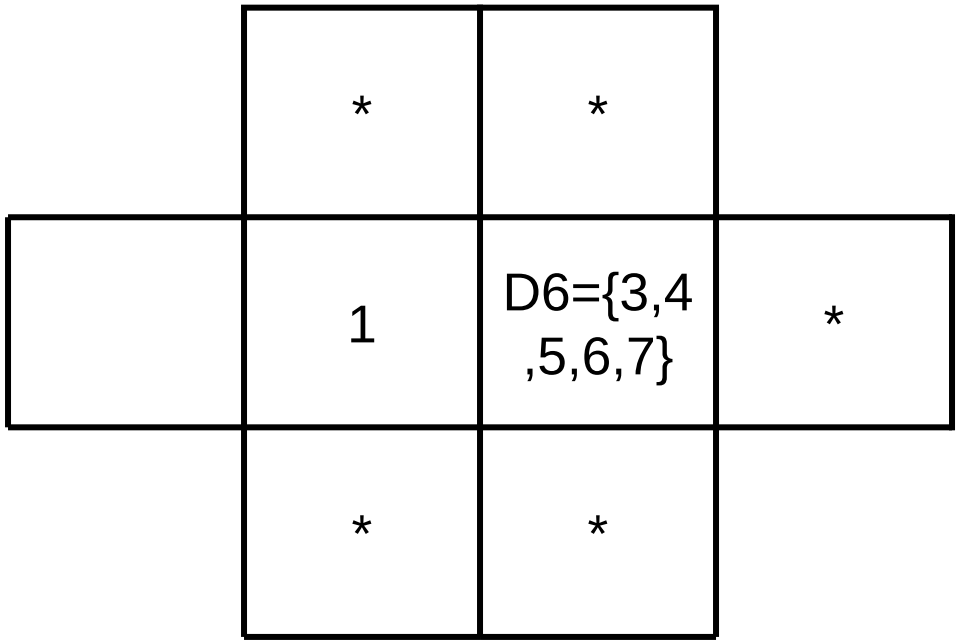
6 “*” squares |D* | = 5

Alldiff constraint => cannot be satisfied

D3 = {1,8}

2.c Is the network consistent?

D6 = {3,4,5,6,7,8}



D* is subset of {2,3,4,5,6,7,8} – {S6-1, S6, S6+1}

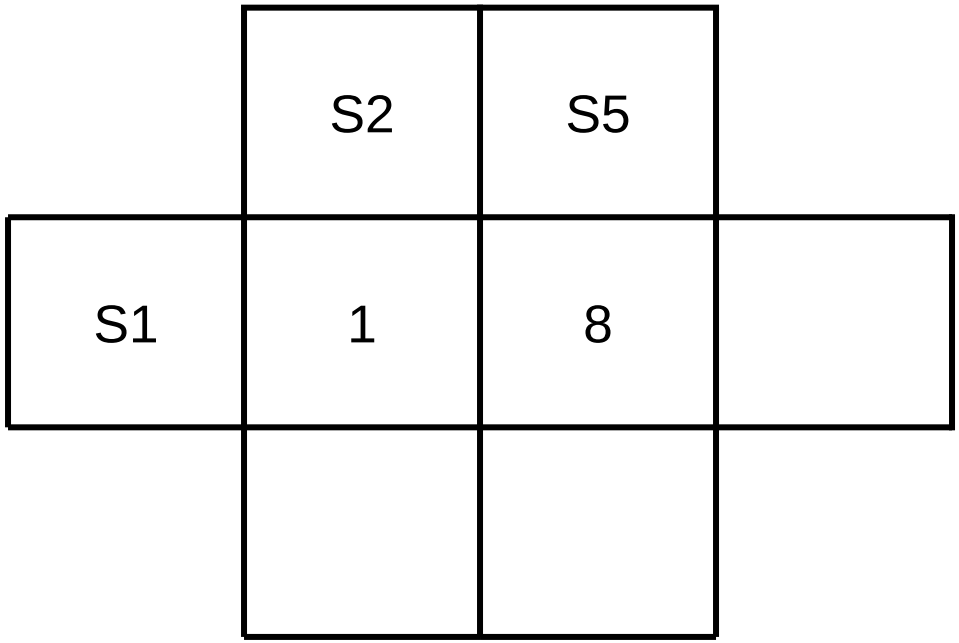
5 “*” squares |D* | <= 4

Alldiff constraint => cannot be satisfied

D6 = {8}

2.c Is the network consistent?

D2 = {3,4,5,6}



S2 = 3 => D5 = {5,6}

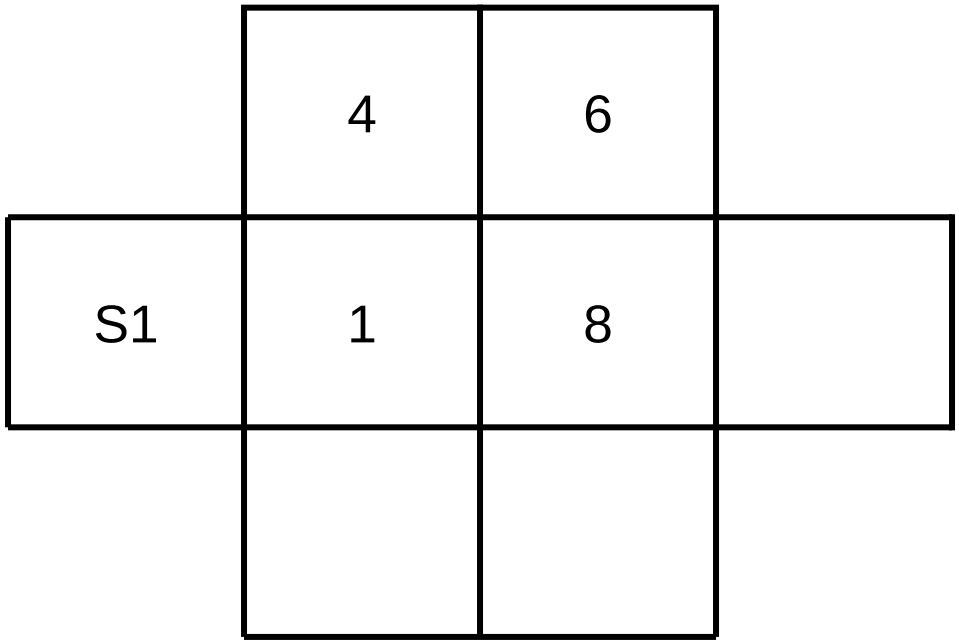
S2 = 4 => D5 = {6}

S2 = 5 => D5 = {3}

S2 = 6 => D5 = {3,4}

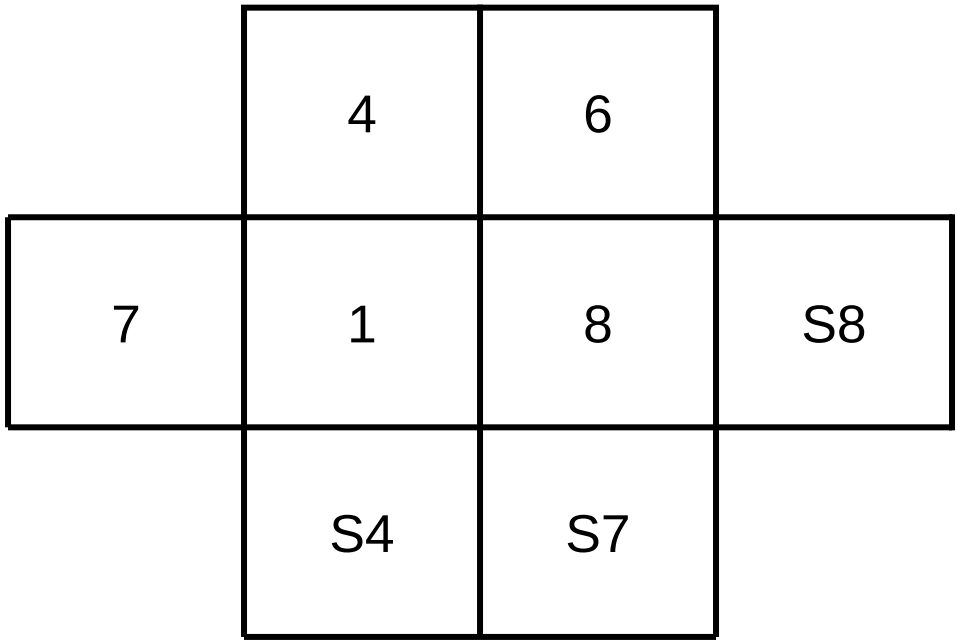
2.c Is the network consistent?

D1= {7}



2.c Is the network consistent?

D4={3,5}, D7={3,5}, D8={2,3}



S7 = 3 => D8 = Null

S7 = 5 => S4 = 3, S8 = 2

2.c Is the network consistent?