- Variables $X_i$ can take values \{0,1,2\}
- Constraints $C_1, C_2, ..., C_5$ enforce the fact that the variables in their arguments must all have different values.
- Unary constraints $C_6, C_7$ enforce:
  
  $C_6$: $X_1 \neq 0$
  $C_7$: $X_4 \neq \{1,2\}$

In the following you must always use the “minimum remaining values heuristic” (MRV) to choose a variable. Use the “degree heuristic” (DH) as a tie-breaker. To assign a value you must use the least constraining value heuristic (LCV).

1. Write out the domains of all variables.

   $D_1 = D_2 = D_3 = D_4 = D_5 = \{0, 1, 2\}$

   Use the unary constraints to simplify the domains of $X_1$, $X_4$

   After simplification of domains of $X_1$ and $X_4$:

   $D_1 = \{1, 2\}$, $D_4 = \{0\}$, $D_2 = D_3 = D_5 = \{0, 1, 2\}$

2. Use MRV & DH to choose the first variable and assign it a value.

   Minimum remaining values (MRV) heuristic - choose the variable with the fewest legal values.

   Domain of $X_4$ has only one value ($D_4 = \{0\}$), so first variable will be $X_4 = 0$. 

3. Use forward checking to simplify the domains of the “neighbors” of that variable. (Neighbors are all variables that share some constraint with that variable)

Neighbors of X4 are: X2 and X5, and their initial domains are D2 = D5 = {0, 1, 2}. For X4 = 0, and constraints X2 ≠ X4 and X5 ≠ X4, we can simplify the domains of X2 and X5 to: D2 = D5 = {1, 2}

4. Simplify the constraint graph by eliminating the first assigned variable from it. Draw the graph as a “standard constraint graph” (without the constraint boxes), where edges represent constraints between pairs of nodes.

![Constraint Graph](image)

Constraints: X1 ≠ X2, X1 ≠ X3, X2 ≠ X3, X2 ≠ X5

5. Use MRV & DH to choose the next variable and assign it a value.

D1 = D2 = D5 = {1, 2}, D3 = {0, 1, 2}

Variables X1, X2, and X5 have the fewest legal values. We will use the “degree heuristic” as a tie-breaker. Degree of node X1 is 2, degree of node X2 is 3 and degree of node X5 is 1. Therefore, we will choose variable X2, and we will assign value X2 = 1 to this variable.

6. Make the entire graph arc-consistent. Provide a solution to the CSP.

From set of constraints: X1 ≠ X2, X1 ≠ X3, X2 ≠ X3, X2 ≠ X5, and set of domains: D1 = D5 = {1, 2}, D3 = {0, 1, 2}, and value X2 = 1, we can find:

X1 = 2; X3 = 0; X5 = 2

7. Express the time complexity of arc-consistency in terms of d (number of states) and n (number of nodes) for general graphs. Is arc-consistency exponential or polynomial in d,n?

O(n^2d^3) – Polynomial in d,n.

8. Same as 7 but now for a tree-structured constraint graph.

O(nd^3) – Polynomial in d,n.