CompSci 171: Intro AI

Homework 5

Games
1.a Tic-Tac-Toe – how many states?

Assume we always start with X:

Empty: 0
1X: \( C(9,1) = 9 \)
1X1O: \( C(9,1)C(8,1) = 72 \)
2X1O: \( C(9,2)C(7,1) = 252 \)
2X2O: \( C(9,2)C(7,2) = 756 \)
3X2O: \( C(9,3)C(6,2) = 1260 \)
3X3O: \( C(9,3)C(6,3) = 1680 \)
4X3O: \( C(9,4)C(5,3) = 1260 \)
4X4O: \( C(9,4)C(5,4) = 630 \)
5X4O: \( C(9,5)C(4,4) = 126 \)

Total:
\[ 1 + 9 + 72 + 252 + 756 + 1260 + 1680 + 1260 + 630 + 126 = 6046 \]

(This includes invalid board positions, e.g. a board with a row of ‘X’ and another of ‘O’)

1.b Tic-Tac-Toe – depth of the complete game tree?

1st level: position of first X
2nd level: position of first Y
3rd level: position of second X
4th level: position of second Y

... Complete game: depth 9

Contains all the valid board positions from (a), assuming we always start with X
1.c Tic-Tac-Toe – game tree (depth 2)?
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Depth 1

Depth 2
1.c Tic-Tac-Toe – game tree (depth 2)?
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1.c Tic-Tac-Toe – game tree (depth 2)?
1.3 Tic-Tac-Toe – game tree (depth 2)?
1.c Tic-Tac-Toe – game tree (depth 2)?
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1.c Tic-Tac-Toe – game tree (depth 2)?
1.d Tic-Tac-Toe – Evaluation of the positions at level 2

The linear evaluation function:

$$3 \times X_2 + X_1 - (3 \times O_2 + O_1)$$

For tree of depth 2, there is one X and one O on the board. The linear evaluation function:

$$X_1 - O_1$$
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1

Depth 2

X=2
Y=1
F=2-1 =1

F=2-2
F=0
F=1

F=2-1
F=1

F=2-3
F=-1

F=3-2
F=0
F=1

F=2-2
F=0
F=1

F=3-2
F=0
1.d Tic-Tac-Toe – game tree (depth 2)?
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1

(3)

Depth 2

F=0  F=1  F=1  F=-1  F=0  F=0  F=1  F=0
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1

Depth 2

F=-1  F=0  F=-1  F=-2  F=0  F=-1  F=0  F=-1
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1
(5)

Depth 2

F=1    F=2    F=1    F=2    F=2    F=1    F=2    F=1
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1

Depth 2

F=-1  F=0  F=-1  F=0  F=-2  F=-1  F=0  F=-1
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1

(7)

Depth 2

F=0     F=1     F=0     F=1     F=-1     F=1     F=1     F=0
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1
(8)

Depth 2

F=-1  F=0  F=-1  F=0  F= -2  F=-1  F=-1  F=-1
1.d Tic-Tac-Toe – game tree (depth 2)?

Depth 1
(9)

Depth 2

F=0  F=1  F=0  F=1  F= -1  F=1  F=0  F=1
1.d Tic-Tac-Toe – game tree (depth 2)?
1.e Tic-Tac-Toe – pruned subtrees

```
1  -1  0  -1  1  -1  0  -1  0
0  -1  1  0  2  0  1  0  1
1  1  1  -1  1  -1  0  -1  0
-1 -2 -1 -2  2  0  1  0  1
1  0  0  0  2 -2 -1 -2 -1
0  1  0 -1  1 -1  1 -1  1
1  0  1  0  2  0  1 -1  0
0  1  0 -1  1  1  0 -1  1
```
1.e Tic-Tac-Toe – pruned subtrees (L-R)
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1.e Tic-Tac-Toe – pruned subtrees (L-R)
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### 1.e Tic-Tac-Toe – pruned subtrees (L-R)

```
1  1  1  1  1
0 -1 -1 -1 -1
1  1  1  1  1
-1 -2 -2 -2 -2
1  1  1  1  1
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
```

```
-1 0 -1 1 0
0 -1 -1 0 0
1 1 1 1 -1
0 0 0 0 0
0 0 0 0 0
```

```
1 0 -1 2 1
-1 2 0 1 0
0 -1 2 0 1
2 2 -2 -1 -1
1 -1 1 -1 1
```

```
0 0 0 0 0
1 1 1 1 -1
0 0 0 0 0
```

```
1 0 -1 1 0
0 -1 -1 0 0
1 1 1 1 -1
0 0 0 0 0
```

```
-1 1 1 1 1
0 -1 0 0 0
1 1 1 1 -1
0 0 0 0 0
```

```
1 0 -1 2 1
-1 2 0 1 0
0 -1 2 0 1
2 2 -2 -1 -1
1 -1 1 -1 1
```

```
0 0 0 0 0
1 1 1 1 -1
0 0 0 0 0
```

```
1 0 -1 1 0
0 -1 -1 0 0
1 1 1 1 -1
0 0 0 0 0
```

```
-1 1 1 1 1
0 -1 0 0 0
1 1 1 1 -1
0 0 0 0 0
```
1.e Tic-Tac-Toe – pruned subtrees (L-R)
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1.e Tic-Tac-Toe – pruned subtrees (L-R)
1.e Tic-Tac-Toe – pruned subtrees (L-R)
1.e Tic-Tac-Toe – pruned subtrees (R-L)
1.e Tic-Tac-Toe – pruned subtrees (R-L)
1.e Tic-Tac-Toe – pruned subtrees (R-L)

```
1   -1   0   -1   1   -1   0   -1
0   -1   1   0   2   0   1   0
1   1   1   -1   1   -1   0   1
-1   -2   -1   -2   2   0   1   0
1   0   0   0   2   -2   -1   -1
0   1   0   -1   1   -1   1   -1
1   0   1   0   2   0   1   0
0   1   0   -1   1   1   0   -1
```
1.e Tic-Tac-Toe – pruned subtrees (R-L)
1.e Tic-Tac-Toe – pruned subtrees (R-L)
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1.e Tic-Tac-Toe – pruned subtrees (R-L)
1. Tic-Tac-Toe – pruned subtrees (R-L)
1.f Tic-Tac-Toe – Property of leaf values

The leaf values should be ordered from lowest to highest from left to right.
2. Min-Max

```
MAX

MIN

MAX

L  M  N  O  P  Q  R  S  T  U  V  W  X  Y
(2) (3) (8) (5) (7) (6) (0) (1) (5) (2) (8) (4) (10) (2)
```
2.a First (MAX) player choice

Max: D = 8
2.b Which nodes would not need to be examined
2.b Which nodes would not need be examined

Max: 3
Min: 3
2.b Which nodes would not need be examined

Min: 8 > 3

Max: 8

Min: 3

Max: 3

Max: 8

Min: 2

Max: 3

Min: 7

Max: 5

Min: 6

Max: 0

Min: 1

Max: 5

Min: 2

Max: 8
2.b Which nodes would not need to be examined.
2.b Which nodes would not need be examined

Min: 7 > 3
Max: 7
2.b Which nodes would not need be examined

Max:

Min:

3 > 3
7 > 3
2.b Which nodes would not need be examined
2.b Which nodes would not need be examined

\[
\begin{align*}
\text{Max:} & \quad 3 > 1 \\
\text{Min:} & \quad 3
\end{align*}
\]
2.b Which nodes would not need be examined
2.b Which nodes would not need to be examined

Max: 3 > 1
Min: 3

Diagram:

- Node A: 3
- Node B: 3
- Node C: 1
- Node D: Max:
- Node E: 2
- Node F: 3
- Node G: 5
- Node H: 7
- Node I: 5
- Node J: 8
- Node K: 10
- Node L: 2
- Node M: 3
- Node N: 8
- Node O: 5
- Node P: 6
- Node Q: 0
- Node R: 1
- Node S: 1
- Node T: 5
- Node U: 2
- Node V: 8
- Node W: 4
- Node X: 10
- Node Y: 2
2.b Which nodes would not need be examined
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2.b Which nodes would not need be examined
2.b Which nodes would not need be examined

Max:

Min:
2.b Which nodes would not need be examined