CompSci 171: Intro AI

Homework 7

First-Order Logic

8.3

- Is the sentence $\exists x,y\ x=y$ valid?

Valid.

An existentially quantified sentence is true in a model if it holds under any extended interpretation in which its variables are assigned to domain elements.

According to the standard semantics of FOL as given in the chapter, every model contains at least one domain element.

Hence, for any model, there is an extended interpretation in which x and y are assigned to the first domain element. In such an interpretation, $x=y$ is true.

8.6

Represent the following sentences in first order logic, using a consistent vocabulary

<u>Vocabulary:</u>

Student(x), Person(x), Man(x), Barber(x), Expensive(x), Agent(x), Insured(x), Smart(x),Politician(x): predicates satisfied by members of the corresponding categories

F, G: French and German courses

x > y: x is greater than y;

Take(x, c, s): student x, course c, semester s

Pass(x, c): student x passes course c

Score(x, c): the score obtained by student x in course c in semester s;

Subject(c, f): the subject of course c is field f;

8.6

Buys(x, y, z): x buys y from z

Sells(x, y, z): x sells y to z

Shaves(x, y): person x shaves person y

Parent(x, y): x is a parent of y

Citizen(x, c, r): x is a citizen of country c for reason r

Resident(x, c): x is a resident of country c

Birthplace(x, u): person x born in country u

Citizen(x, u): person x is a citizen of country u

Parent(x, z): z is a parent of x

Fools(x, y, t): person x fools person y at time t

8.6

a) Some students took French in spring 2001.

$\exists x\ Student(x) \wedge Takes(x, F, Spring2001).$

b) Every student who takes French passes it.

$\forall\ x, s\ Student(x) \wedge Takes(x, F, s) \Rightarrow Passes(x, F, s).$

c) Only one student took Greek in spring 2001.

$\exists x\ Student(x) \wedge Takes(x,G, Spring2001) \wedge \forall y\ y \neq x \Rightarrow \neg Takes(y,G,Spring2001).$

d) The best score in Greek is always higher than the best score in French.

$\forall s\ \exists x\ \forall y\ Score(x,G,s) > Score(y,F, s).$

e) Every person who buys a policy is smart.

$\forall x\ Person(x) \wedge (\exists y, z\ Policy(y) \wedge Buys(x, y, z)) \Rightarrow Smart(x).$

f) No person buys an expensive policy.

$\forall\ x, y, z\ Person(x) \wedge Policy(y) \wedge Expensive(y) \Rightarrow \neg Buys(x, y, z).$

8.6

g) There is an agent who sells policies only to people who are not insured.

$\exists x\ Agent(x) \land \forall y, z\ Policy(y) \land Sells(x, y, z) \Rightarrow (Person(z) \land \neg Insured(z)).$

h) There is a barber who shaves all men in town who do not shave themselves.

$\exists x\ \forall y\ Barber(x) \land Man(y) \land \neg Shaves(y, y) \Rightarrow Shaves(x, y).$

i) A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

$\forall x\ Person(x) \land Born(x,UK) \land (\forall y\ Parent(y, x) \Rightarrow ((\exists\ r\ Citizen(y,UK, r)) \lor$
$Resident(y,UK))) \Rightarrow Citizen(x,UK,Birth).$

j) A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

$\forall x\ Person(x) \land \neg Born(x,UK) \land (\exists y\ Parent(y, x) \land Citizen(y,UK,Birth))$
$\Rightarrow Citizen(x,UK,Descent).$

8.6

k) Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can’t fool all of the people all of the time.

$\forall$ x Politician(x) $\Rightarrow$ ($\exists$ y $\forall$t Person(y) $\wedge$ Fools(x, y, t)) $\wedge$ ($\exists$ t $\forall$y Person(y) $\Rightarrow$ Fools(x, y, t)) $\wedge \neg$($\forall$ t $\forall$y Person(y) $\Rightarrow$ Fools(x, y, t))

8.7

- Represent the sentence “All Germans speak the same languages” in predicate calculus.

$$\forall x, y, l \;\; German(x) \land German(y) \land Speaks(x, l) \;\Rightarrow\; Speaks(y, l)$$

8.8

- What axiom is needed to infer the fact *Female(Laura)* given the facts *Male(Jim)* and *Spouse(Jim, Laura)*

$$\forall x, y \quad Spouse(x, y) \land Male(x) \;\Rightarrow\; Female(y)$$

8.15

- Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:

$$\forall x, y \;\; Adjacent([x, y], [x + 1, y]) \wedge Adjacent([x, y], [x, y + 1])$$

There are several problems with the proposed definition.

  - It allows one to prove, say *Adjacent*([1,1],[1,2]) but not *Adjacent*([1,2],[1,1]); so we need an additional symmetry axiom.
  - It does not allow one to prove that *Adjacent*([1,1],[1,3]) is false, so it needs to be written as

$$\forall s_1, s_2 \quad \Leftrightarrow \quad \ldots$$

  - Finally, it does not work as the boundaries of the world, so some extra conditions must be added