

CompSci 171: Intro AI

Homework 8

Inference in FOL

9.3

9.3 Suppose a knowledge base contains just one sentence, $\exists x \text{ AsHighAs}(x, \text{Everest})$. Which of the following are legitimate results of applying Existential Instantiation?

- a. $\text{AsHighAs}(\text{Everest}, \text{Everest})$.
- b. $\text{AsHighAs}(\text{Kilimanjaro}, \text{Everest})$.
- c. $\text{AsHighAs}(\text{Kilimanjaro}, \text{Everest}) \wedge \text{AsHighAs}(\text{BenNevis}, \text{Everest})$
(after two applications).

Both b and c are valid

a is invalid because it introduces the previously-used symbol *Everest*. Note that c does not imply that there are two mountains as high as Everest, because nowhere is it stated that *BenNevis* is different from *Kilimanjaro* (or *Everest*, for that matter).

9.4

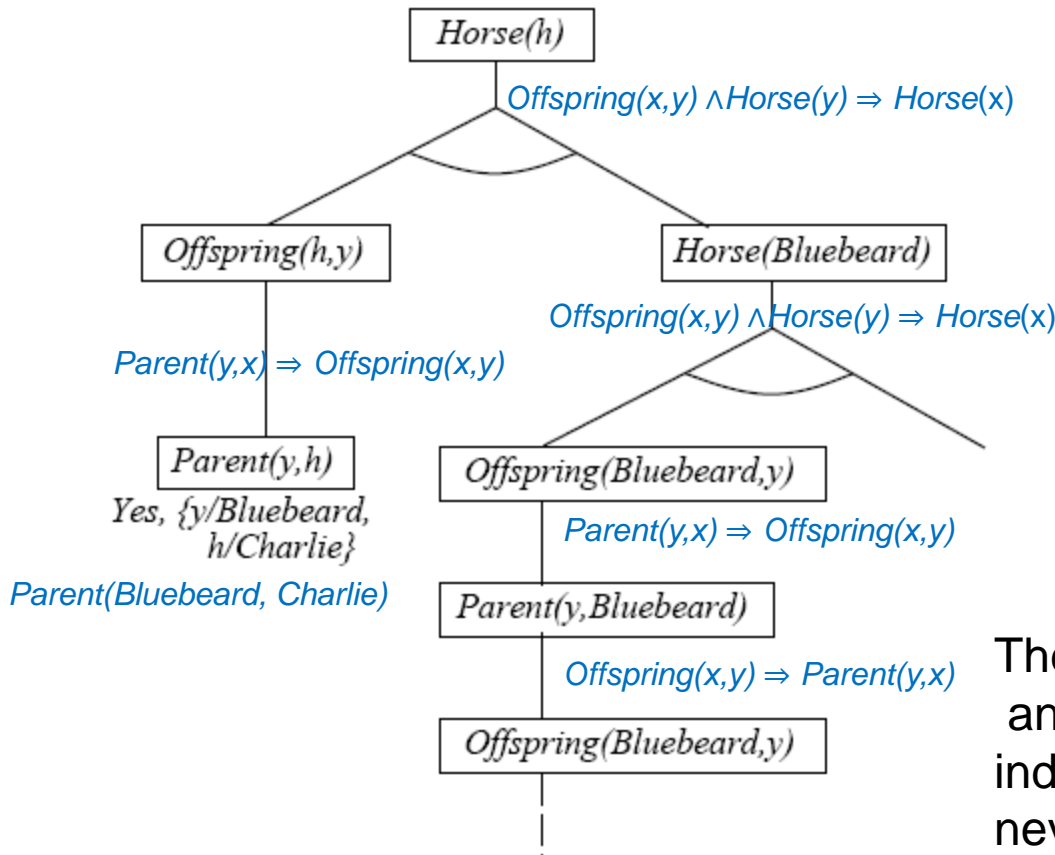
- a. $P(A,B,B), P(x,y,z)$
 - $\{x/A, y/B, z/B\}$
- b. $Q(y,G(A,B)), Q(G(x,x),y)$
 - No unifier ($\{y/G(A,B), G(A,B)/G(x,x)\}$)? x cannot bind to both A and B .
- c. $Older(Father(y),y), Older(Father(x), John)$
 - $\{y/John, x/John\}$.
- d. No unifier ($\{x/y, y/Father(y)\}$)? the occurs-check prevents unification of y with $Father(y)$.

9.9

- a. Horse, cows, and pigs are mammals.
 - $Horse(x) \Rightarrow Mammal(x)$
 - $Cow(x) \Rightarrow Mammal(x)$
 - $Pig(x) \Rightarrow Mammal(x)$
- b. An offspring of a horse is a horse.
 - $Offspring(x,y) \wedge Horse(y) \Rightarrow Horse(x)$
- c. Bluebeard is a horse.
 - $Horse(Bluebeard)$
- d. Bluebeard is Charlie's parent.
 - $Parent(Bluebeard, Charlie)$
- e. Offspring and parent are inverse relations.
 - $Offspring(x,y) \Rightarrow Parent(y,x)$
 - $Parent(y,x) \Rightarrow Offspring(x,y)$
- f. Every mammal has a parent.
 - $Mammal(x) \Rightarrow Parent(G(x),x)$

9.10

Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $\exists h \text{ Horse}(h)$, where clauses are matched in the order given.



$Horse(x) \Rightarrow Mammal(x)$
 $Cow(x) \Rightarrow Mammal(x)$
 $Pig(x) \Rightarrow Mammal(x)$
 $Offspring(x,y) \wedge Horse(y) \Rightarrow Horse(x)$
 $Horse(Bluebeard)$
 $Parent(Bluebeard, Charlie)$
 $Offspring(x,y) \Rightarrow Parent(y,x)$
 $Parent(y,x) \Rightarrow Offspring(x,y)$
 $Mammal(x) \Rightarrow Parent(G(x),x)$

The branch with $Offspring(Bluebeard,y)$ and $Parent(y,Bluebeard)$ repeats indefinitely, so the rest of the proof is never reached!

9.18 (a)

- Horse are animals
 - $\forall x \text{ Horse}(x) \Rightarrow \text{Animal}(x)$
- The head of a horse is the head of an animal
 - $\forall x, h \text{ Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \text{ Animal}(y) \wedge \text{HeadOf}(h,y)$

9.18 (b)

- $\forall x \text{ Horse}(x) \Rightarrow \text{Animal}(x) \equiv \neg \text{Horse}(x) \vee \text{Animal}(x)$
- $\forall x, h \text{ Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \text{ Animal}(y) \wedge \text{HeadOf}(h,y)$
 - $\neg(\forall x,h \text{ Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \text{ Animal}(y) \wedge \text{HeadOf}(h,y))$
 - $\neg(\forall x,h \neg(\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \vee (\exists y \text{ Animal}(y) \wedge \text{HeadOf}(h,y)))$
 - $\exists x,h (\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \wedge \neg(\exists y \text{ Animal}(y) \wedge \text{HeadOf}(h,y))$
 - $\exists x,h (\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \wedge (\forall y \neg \text{Animal}(y) \vee \neg \text{HeadOf}(h,y))$
 - $(\text{Horse}(G) \wedge \text{HeadOf}(H,G)) \wedge (\neg \text{Animal}(y) \vee \neg \text{HeadOf}(H,y))$
 - $\text{Horse}(G)$
 $\wedge \text{HeadOf}(H,G)$
 $\wedge \neg \text{Animal}(y) \vee \neg \text{HeadOf}(H,y)$

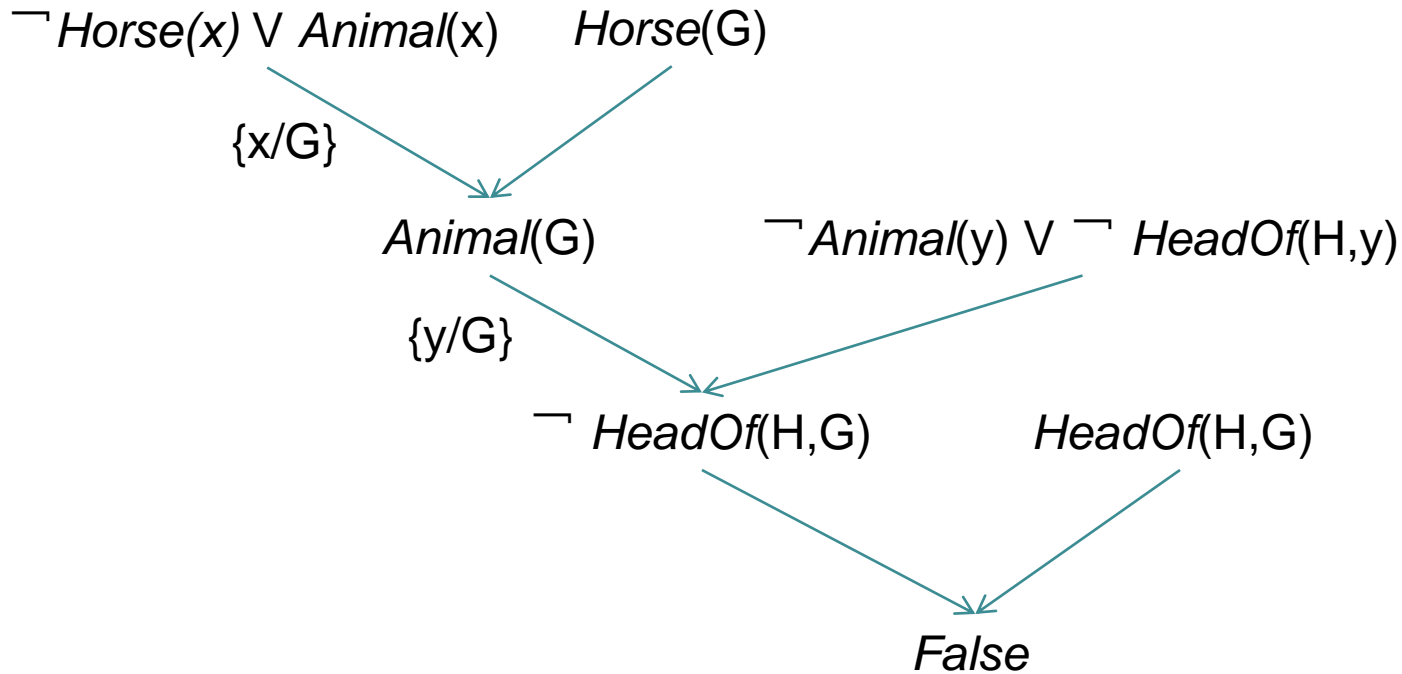
A. $\neg \text{Horse}(x) \vee \text{Animal}(x)$

B. $\text{Horse}(G)$

C. $\text{HeadOf}(H,G)$

D. $\neg \text{Animal}(y) \vee \neg \text{HeadOf}(H,y)$

9.18 (c)



9.19 (a)

- $\forall x \exists y (x \geq y)$
 - For every natural number there is some other natural number that is smaller than or equal to it
- $\exists y \forall x (x \geq y)$
 - There is a particular natural number that is smaller than or equal to any natural number

9.19 (b, c)

- A. For every natural number there is some other natural number that is smaller than or equal to it
- Yes, (A) is true under this interpretation. You can always pick the number itself for the “some other” number.
- B. There is a particular natural number that is smaller than or equal to any natural number
- Yes, (B) is true under this interpretation. You can pick 0 for the “particular natural number.”

9.19 (d, e)

- A. For every natural number there is some other natural number that is smaller than or equal to it
- B. There is a particular natural number that is smaller than or equal to any natural number

d. No, (A) does not logically entail (B).

e. Yes, (B) logically entails (A). You can always pick that particular number in (B) as some other natural number in (A)

9.19 (f)

$$(A) \forall x \exists y (x \geq y)$$

$$\rightarrow \forall x (x \geq F_1(x))$$

$$\rightarrow x \geq F_1(x)$$

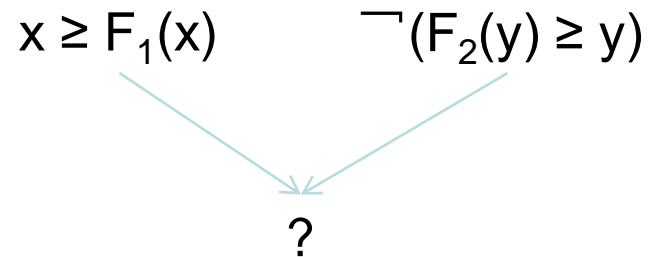
$$(\neg B) \exists y \forall x (x \geq y)$$

$$\rightarrow \neg(\exists y \forall x (x \geq y))$$

$$\rightarrow \forall y \exists x \neg(x \geq y)$$

$$\rightarrow \forall y \neg(F_2(y) \geq y)$$

$$\rightarrow \neg(F_2(y) \geq y)$$



- Unification of $x \geq F_1(x)$ and $\neg(F_2(y) \geq y)$:
 - $\{x/F_2(y), F_1(F_2(y))/y\}$ fails in the occurs check

9.19 (g)

$$(\neg A) \forall x \exists y (x \geq y)$$

$$\rightarrow \neg(\forall x \exists y (x \geq y))$$

$$\rightarrow \exists x \forall y \neg(x \geq y)$$

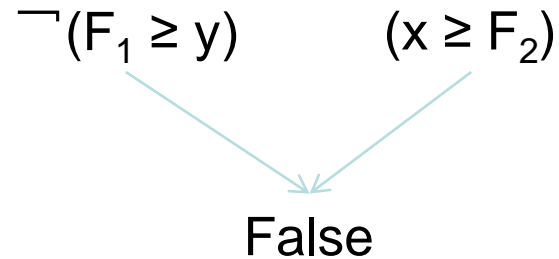
$$\rightarrow \forall y \neg(F_1 \geq y)$$

$$\rightarrow \neg(F_1 \geq y)$$

$$(B) \exists y \forall x (x \geq y)$$

$$\rightarrow \forall x (x \geq F_2)$$

$$\rightarrow (x \geq F_2)$$



- Unification of $\neg(F_1 \geq y)$ and $(x \geq F_2)$:
 - $\{y/F_1, x/F_2\}$