CSP

1) Consider the following binary constraint network: There are 4 variables: $X_1$, $X_2$, $X_3$, $X_4$ with the domains: $D_1=\{1,2,3,4\}$, $D_2=\{3,4,5,8,9\}$, $D_3=\{2,3,5,6,7,9\}$, $D_4=\{3,5,7,8,9\}$. The constraints are $X_1 \geq X_2$, $X_2 > X_3$ or $X_3 - X_2 = 2$, $X_3 \neq X_4$.

   a. Draw the constraint graph.

   b. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final network)

   c. Is the network consistent? If yes, give a solution.

2) Consider the 8 squares positioned as follows:

   The task is to label the boxes above with the numbers 1-8 such that the labels of any pair of adjacent squares (i.e. horizontal, vertical or diagonal) differ by at least 2 (i.e. 2 or more).

   a. Write all constraints and draw the constraint graph.

   b. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final arc-consistent network)

   c. Is the network consistent? If yes, give a solution.
Variables $X_i$ can take values $\{0, 1, 2\}$

- Constraints $C_1, \ldots, C_5$ enforce the fact that the variables in their arguments must all have different values.
- Unary constraints $C_6, C_7$ enforce:
  - $C_6$: $X_1 \rightarrow 0$
  - $C_7$: $X_4 \rightarrow \{1, 2\}$

In the following you must always use the “minimum remaining values values heuristic” (MRV) to choose a variable. Use the “degree heuristic” (DH) as a tie-breaker. To assign a value you must use the least constraining value heuristic (LCV).

1. [2pts] Write out the domains of all variables.
   Use the unary constraints to simplify the domains of $X_1, X_4$.
2. [2pts] Use MRV & DH to choose the first variable and assign it a value.
3. [4pts] Use forward checking to simplify the domains of the “neighbors” of that variable.
   (neighbors are all variables that share some constraint with that variable.)
4. [4pts] Simplify the constraint graph by eliminating the first assigned variable from it.
   Draw the graph as a “standard constraint graph” (without the constraint boxes)
   where edges represent constraints between pairs of nodes.
5. [2pts] Use MRV & DH to choose the next variable and assign it a value.
6. [4pts] Make the entire graph arc-consistent. Provide a solution to the CSP.
7. [4pts] Express the time complexity of arc-consistency in terms of $d$ (number of states)
   and $n$ (number of nodes) for general graphs. Is arc-consistency exponential or polynomial in $d, n$?
8. [2pts] Same as 7 but now for a tree-structured constraint graph.