Homework Games

1) (Based on question 6.1 in Russel and Norvig) Consider a Tic-Tac-Toe game. Let $X_n$ be the number of rows, columns or diagonals containing exactly $n$ X's and no O's. Similarly, let $O_n$ be the number of rows, columns or diagonals containing exactly $n$ O's and no X's. We propose a utility function which assigns +1 to any position with $X_3 = 1$ (i.e. winning position) and assigns -1 to any position with $O_3 = 1$ (i.e. loosing position). The linear evaluation function we suggest is $3X_2 + X_1 - (3O_2 + O_1)$:

a) How many states (i.e. board positions) are there in a Tic-Tac-Toe game (including symmetric board positions)

b) What is the depth of the complete game tree? Does the complete game tree contain all the board positions you counted in (a)?

c) Show the game tree down to depth 2, namely starting from an empty board to all position in which there is one X and one O on the board.

d) Mark on your tree the evaluation of the positions at level 2 (i.e. the value of the utility function at each position). Thereafter, mark on your tree the min-max values.

e) Apply alpha-beta search on your tree, and mark the pruned subtrees when traversing from left to right, from right to left. What is the optimal order?

f) What property should the leaf values of a tree have so that pruning will be maximized (resp. minimized) when the tree below is traversed from left to right.
2) Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player’s point of view. Assume that the first player is the maximizing player.

a) What move should the first player choose?

b) What nodes would not need be examined using the alpha-beta algorithm – assuming that the nodes are examined left-to-right order?
3) Consider the following game tree with MAX nodes (upward pointing triangles), MIN nodes (downward pointing triangles and chance nodes (circles). The leaf nodes represent the profit made by MAX if the game ends up in that leaf node. MIN is a logical agent and tries to minimize the profit of MAX. Chance nodes represent a fair coin, i.e. going left or right has equal probability.

3a) Compute the expected minimax value for each node in the game tree. Write your answer into the figure.

3b) Search the game tree using depth first search (no pruning). Explore branches on the right before you explore left branches. Maintain bounds on the expected minimax value for MIN and MAX nodes (but not for chance nodes). A bound on an expected minimax value means that the average profit for MAX is bounded. It does not mean that the bound is respected in any particular instance of the game. List the nodes in the order that you update their bounds (or their values). Every time a bound or value gets updated you must insert that into the list. Write your answer as a sequence of nodes with their expected minimax value between brackets behind it, e.g. H(4),F(<8), ... This means that a node may appear multiple times, if you update its bound more than once.
3c) The objective is to maximize the expected profit for MAX and we assume that MIN tries to minimize the expected profit for MAX. Which branches could have been pruned in the DFS search of question 1b)?

3d) What are the possible outcomes of playing this game when both players are logical agents and what are the probabilities of these outcomes occurring?

3e) During the first game, MAX collects 2 dollars. MAX is disappointed because he knows he was playing optimally but still failed. Does MAX’s failure mean that he should not consider himself a logical agent? (explain briefly).