1. (a) Since all constraints are pairwise, the box shaped nodes could be omitted.

\[ X_1 \square X_2 \square X_3 \square X_4 \]

(b) No, it is not arc-consistent.

Enforce arc-consistency between \(X_1\) and \(X_2\): \(D_1 = \{3, 4\}\) \(D_2 = \{3, 4\}\)

\(X_2\) and \(X_3\): \(D_2 = \{3, 4\}\) \(D_3 = \{2, 3, 5, 6\}\)

\(X_3\) and \(X_4\): \(D_3 = \{2, 3, 5, 6\}\) \(D_4 = \{3, 5, 7, 8, 9\}\)

So the arc-consistent domains are

\[
D_1 = \{3, 4\}\quad D_2 = \{3, 4\}\quad D_3 = \{2, 3, 5, 6\}\quad D_4 = \{3, 5, 7, 8, 9\}
\]

(c) Yes,

\(X_1 = 4, X_2 = 4, X_3 = 3, X_4 = 9\)

2. (a) Name variables for each square in raster scan order \(X_1\) through \(X_8\). Thus the constraints are \(C_{12} = |X_1 - X_2| \geq 2\), \(C_{14} = |X_1 - X_4| \geq 2\), \(C_{25} = |X_2 - X_5| \geq 2\), \(C_{34} = |X_3 - X_4| \geq 2\), \(C_{45} = |X_4 - X_5| \geq 2\), \(C_{47} = |X_4 - X_7| \geq 2\), \(C_{56} = |X_5 - X_6| \geq 2\), \(C_{58} = |X_5 - X_8| \geq 2\), \(C_{78} = |X_7 - X_8| \geq 2\), and finally the constraint over all variables so that \(X_i \neq X_j\) \(\forall i, j\)

(b) Yes, it is arc-consistent

(c) Yes

\[
\begin{array}{c}
3 - 5 \\
7 - 1 - 8 - 2 \\
4 - 6 \\
\end{array}
\]

3. (a) \(D_1 = \{0\}\), \(D_2 = \{0, 1, 2\}\), \(D_3 = \{0, 1, 2\}\), \(D_4 = \{1, 2\}\), \(D_5 = \{0, 1, 2\}\)

(b) MRV picks \(X_1\) and assigns it to its only possible value, 0.

(c) From \(C_3\) we get \(D_2 = \{1, 2\}\), and \(D_3 = \{1, 2\}\).
(d) $X_2 \leftarrow X_3$
\[ X_4 \leftarrow X_5 \]

(e) MRV gives a tie between $X_2$, $X_3$, and $X_4$, and DH chooses $X_2$. LCV would not prefer either $X_2 = 1$ or $X_2 = 2$, so say $X_2 = 1$.

(f) Arc-consistent domains: $D_1 = \{0\}$, $D_2 = \{1, 2\}$, $D_3 = \{1, 2\}$, $D_4 = \{1, 2\}$, $D_5 = \{0, 1, 2\}$. One solution is $X_1 = 0$, $X_2 = 1$, $X_3 = 2$, $X_4 = 2$, $X_5 = 0$.

(g) Straightforward implementations of arc-consistency give $O(n^2d^3)$. $d^2$ for each arc consistency check, each node can be checked at most $d$ times, and $n^2$ edges in a full graph gives $O(n^2d^3)$. This is polynomial in $d$ and $n$.

(h) For a tree-structured constraint graph, the running time is $O(nd^2)$. $d^2$ for each arc consistency check, each node will be checked at most 2 times (ie propagate from the root to the leaves, then from the leaves to the root), and $n$ nodes in a tree gives $O(nd^2)$. This also is polynomial in $d$ and $n$. 