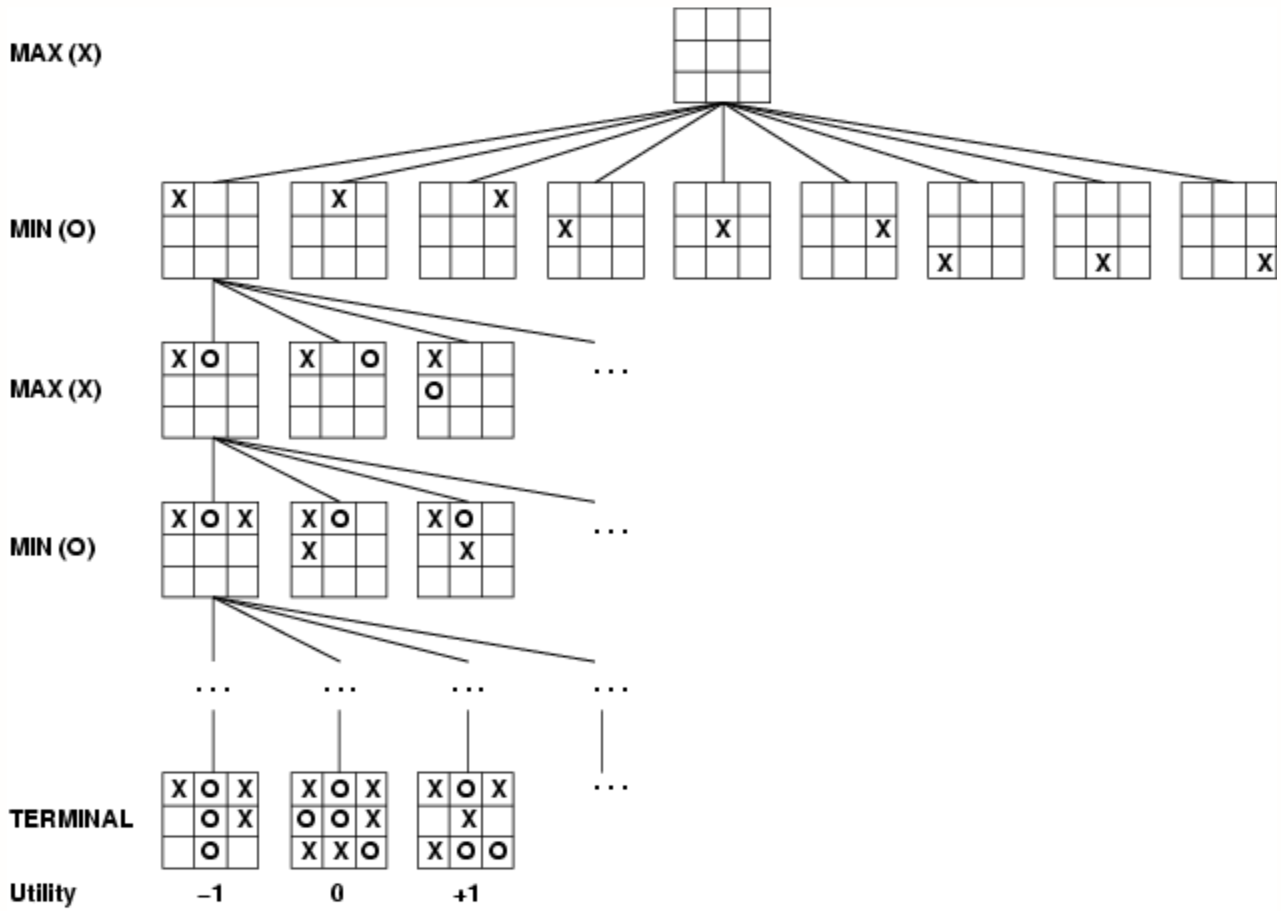
Games & Adversarial Search

Game tree (2-player, deterministic, turns)



How do we search this tree to find the optimal move?

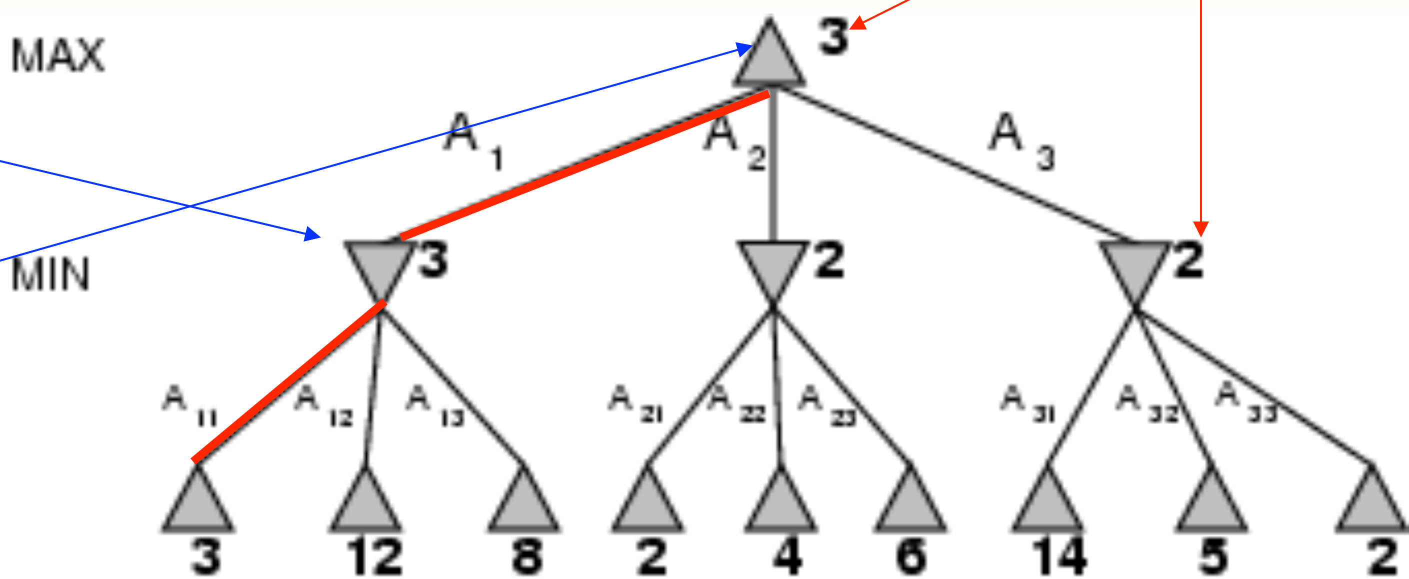
Minimax

- Idea: choose a move to a position with the highest minimax value = best achievable payoff against a rational opponent.

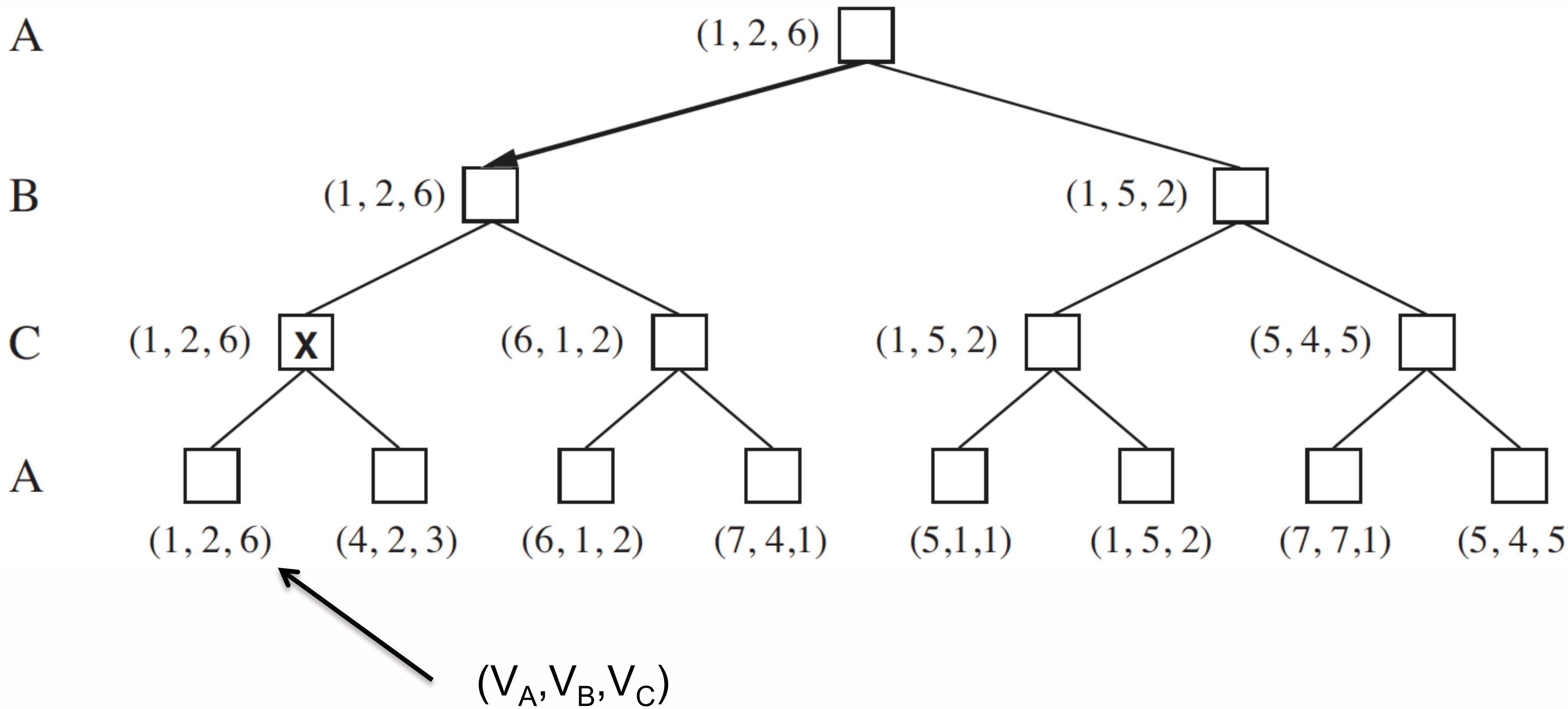
Example: deterministic 2-ply game:

minimax value

Minimax value is computed bottom up:
-Leaf values are given.
-3 is the best outcome for MIN in this branch.
-3 is the best outcome for MAX in this game.
-We explore this tree in depth-first manner.



Multiplayer Games



Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an rational opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

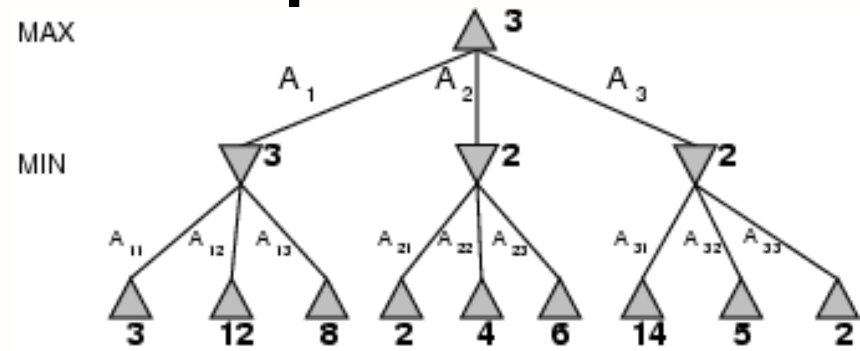
- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible

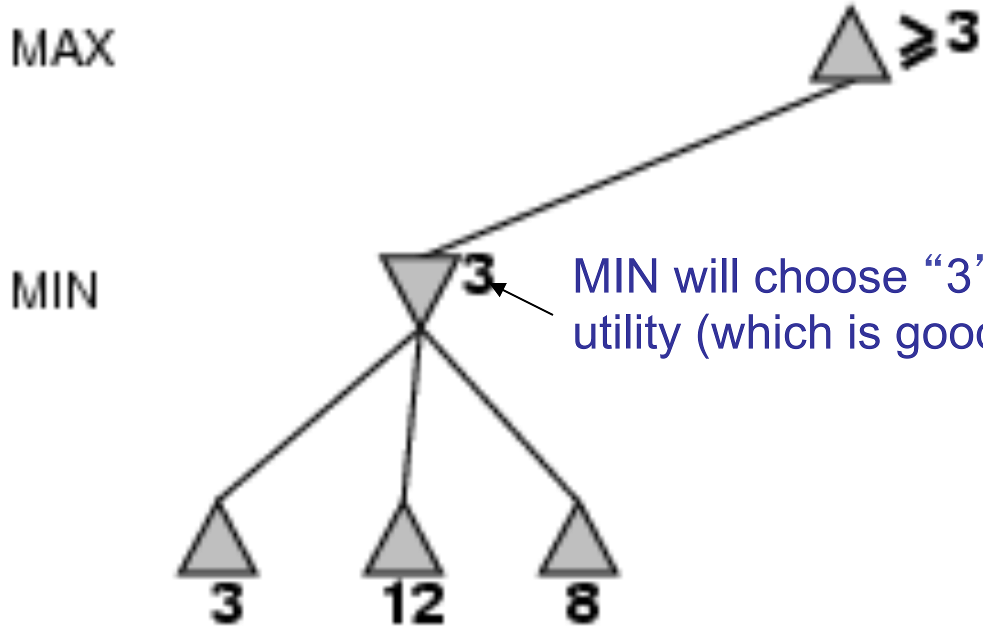
$\alpha - \beta$ Pruning

1. Do we need to expand all nodes?

2. No: We can do better by pruning branches that will not lead to success.

α-β pruning example

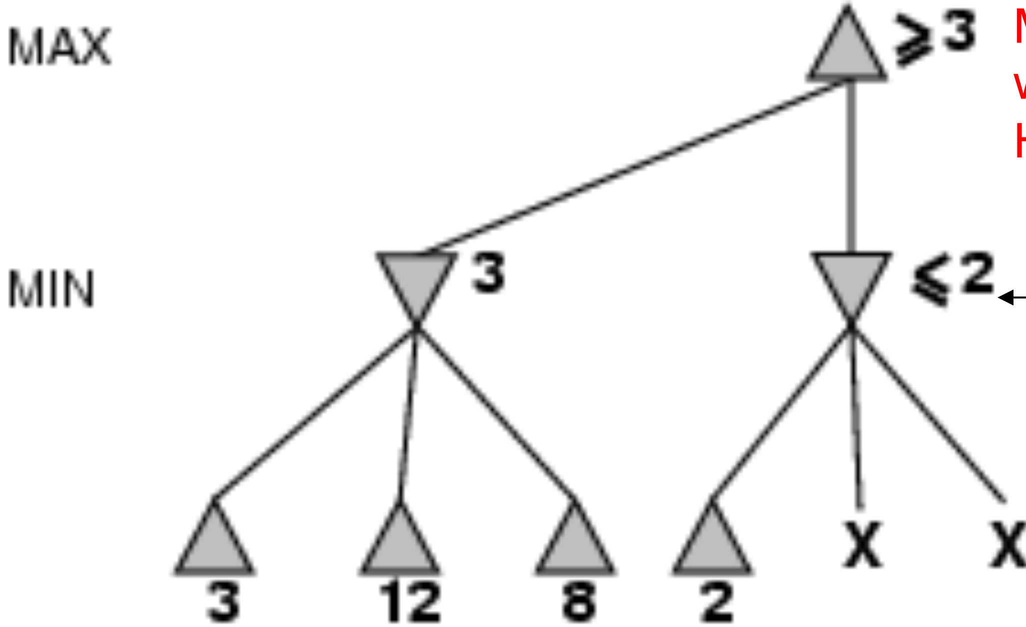




MAX knows that it can at least get "3" by playing this branch

MIN will choose "3", because it minimizes the utility (which is good for MIN)

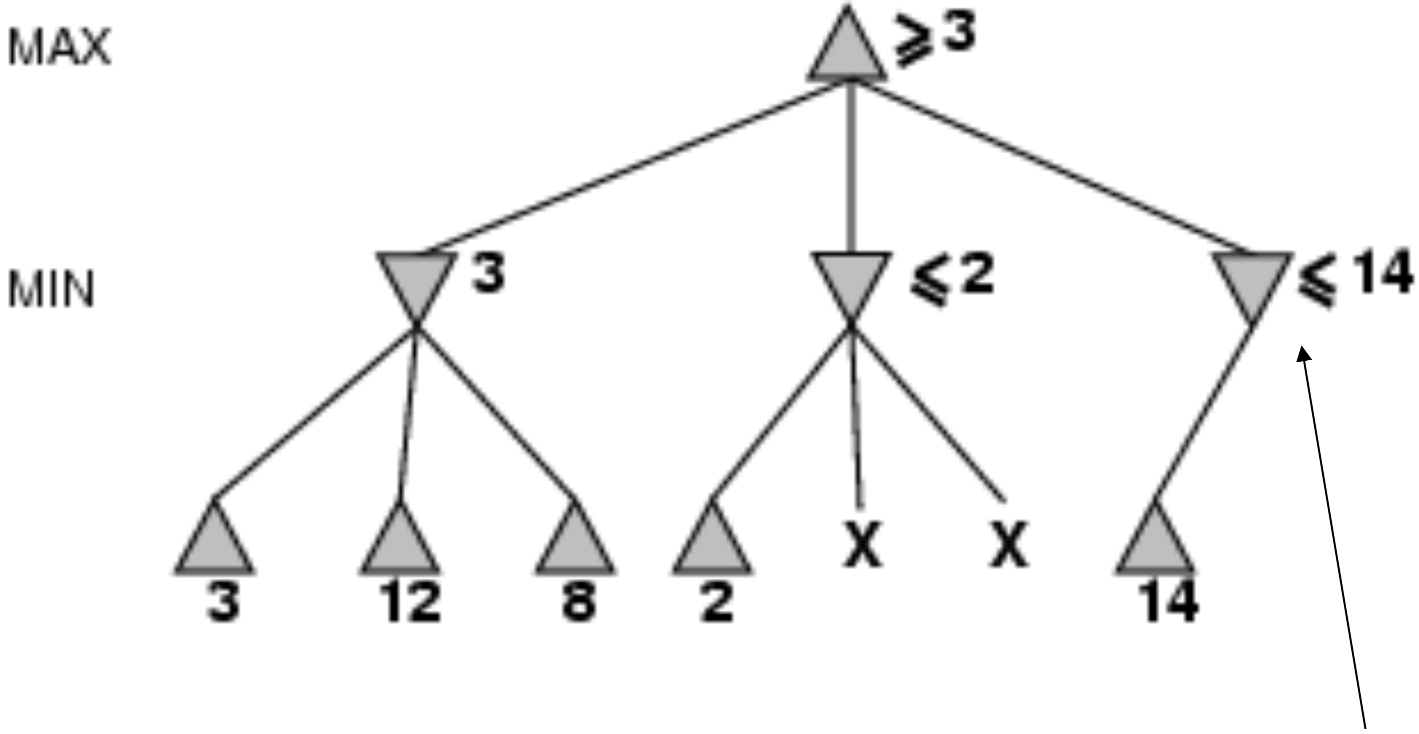
α-β pruning example



MAX knows that the new branch will never be better than 2 for him. He can *ignore* it.

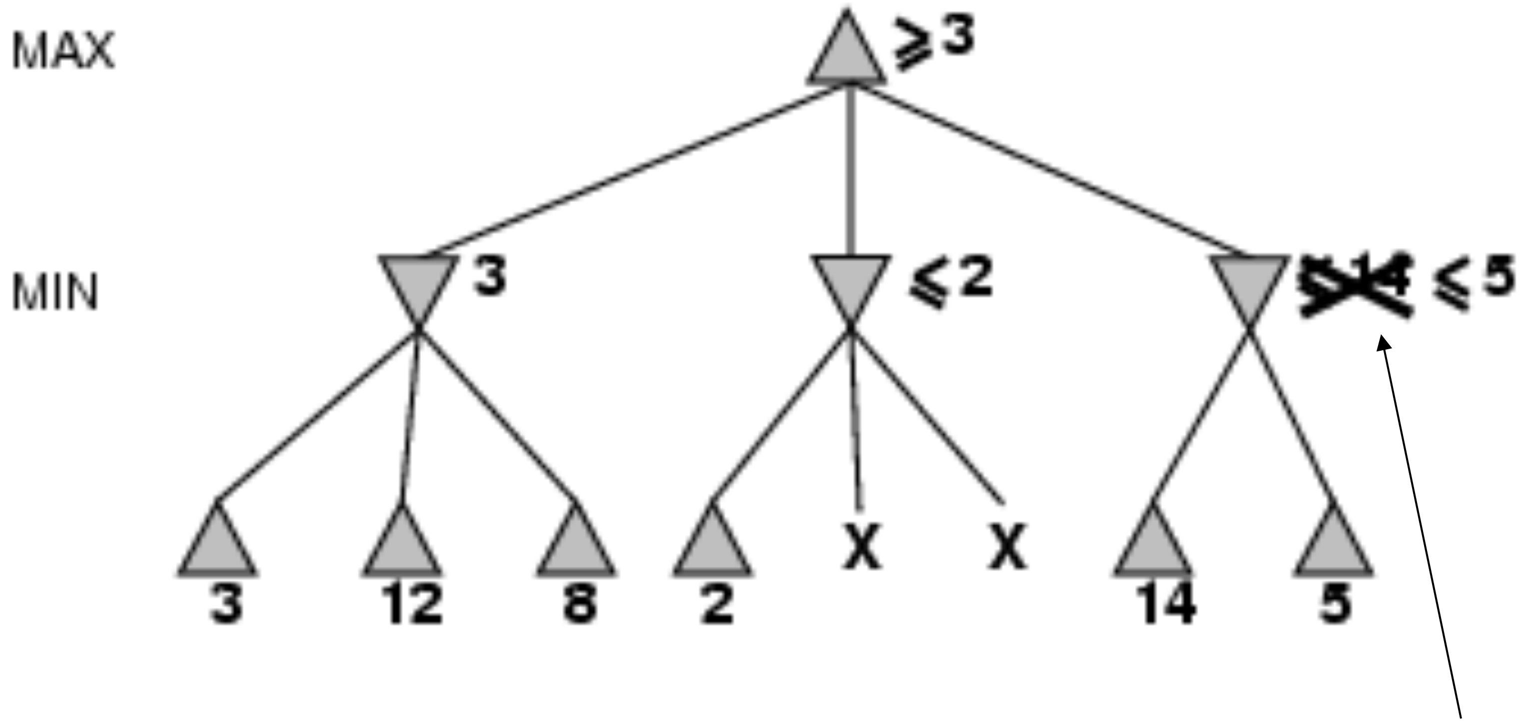
MIN can certainly do as good as 2, but maybe better (= smaller)

α-β pruning example



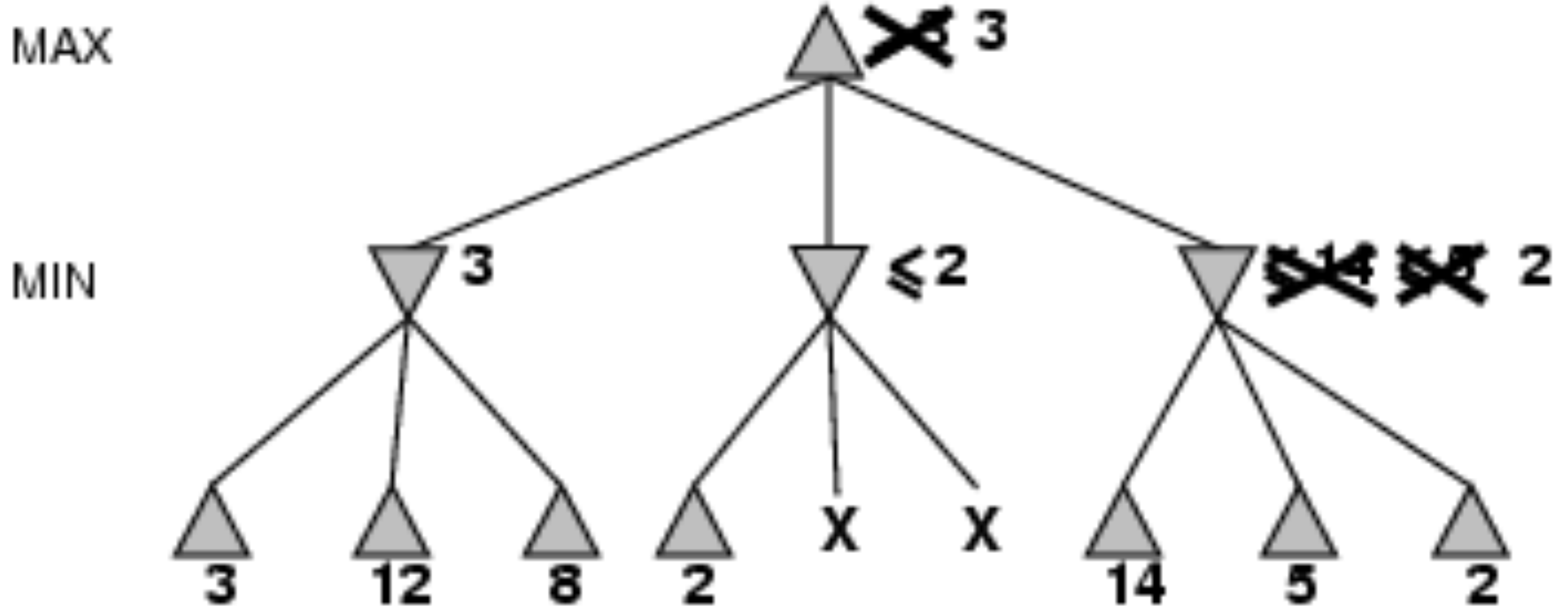
MIN will do at least as good as 14 in this branch (which is very good for MAX!) so MAX will want to explore this branch more.

α-β pruning example



MIN will do at least as good as 5 in this branch (which is still good for MAX) so MAX will want to explore this branch more.

α-β pruning example



Bummer (for MAX): MIN will be able to play this last branch and get 2. This is worse than 3, so MAX will play 3.

Properties of α-β

- Pruning does not affect final result (it is exact).
- Good move ordering improves effectiveness of pruning (see last branch in example).
- Different orderings of sequences of moves may lead to same state. Save the value of these “transpositions” to avoid double work.
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search

The Algorithm

- Visit the nodes in a depth-first manner
- Maintain bounds on nodes.
- A bound may change if one of its children obtains a unique value.
- A bound becomes a unique value when all its children have been checked or pruned.
- When a bound changes into a tighter bound or a unique value, it may become inconsistent with its parent.
- When an inconsistency occurs, prune the sub-tree by cutting the edge between the inconsistent bounds/ values.

→ This is like propagating changes bottom-up in the tree.

Practical Implementation

How do we make this practical?

Standard approach:

- cutoff test: (where do we stop descending the tree)
  - depth limit
  - better: iterative deepening
  - cutoff only when no big changes are expected to occur next (quiescence search).

- evaluation function
  - When the search is cut off, we evaluate the current state by estimating its utility. This estimate is captured by the evaluation function.
  - Run α-β pruning minimax with these estimated values at the leaves instead.

Evaluation functions

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with
  $f_1(s)$ = (number of white queens) – (number of black queens), etc.

Forward Pruning & Lookup

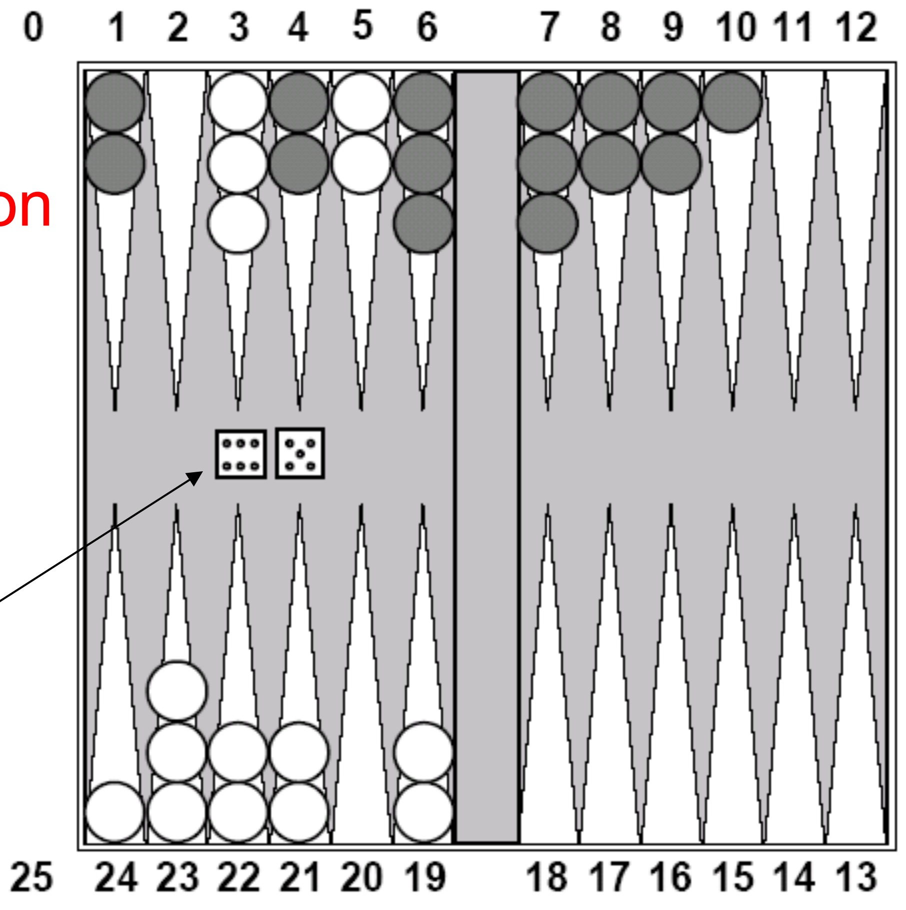
- Humans don't consider all possible moves.
- Can we prune certain branches immediately?
- "ProbCut" estimates (from past experience) the uncertainty in the estimate of the node's value and uses that to decide if a node can be pruned.
- Instead of search one can also store game states.
- Openings in chess are played from a library
- Endgames have often been solved and stored as well.

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994.
- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997.
- Othello: human champions refuse to compete against computers: they are too good.
- Go: human champions refuse to compete against computers: they are too bad.
- Poker: Machine was better than best human poker players in 2008.

Chance Games.

Backgammon

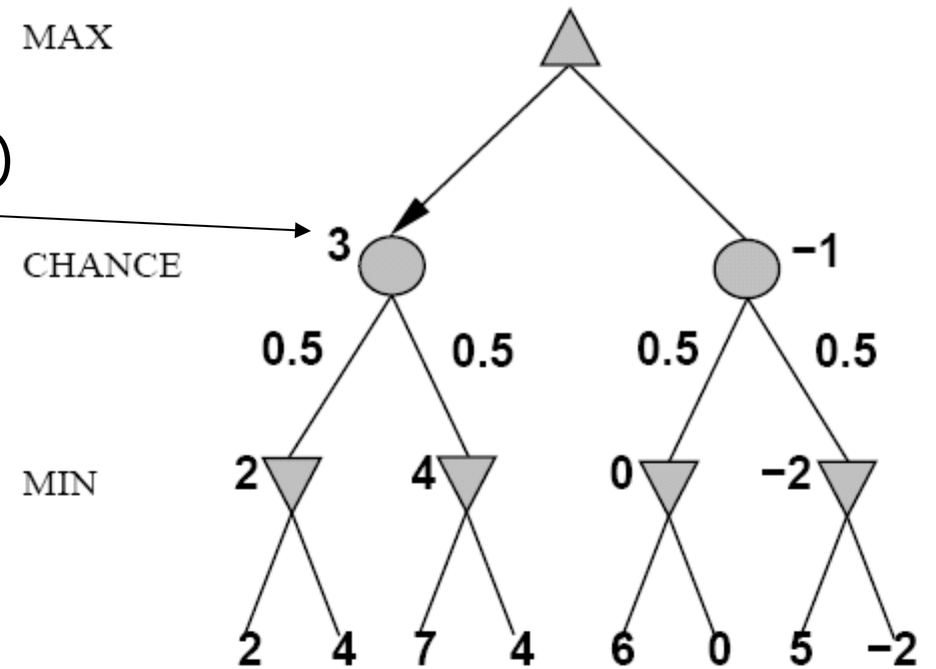


your element of chance

Expected Minimax

$$v = \sum_{\text{chance nodes}} P(n) \times \text{Minimax}(n)$$

$$3 = 0.5 \times 4 + 0.5 \times 2$$



Again, the tree is constructed bottom-up.

Now we have even more nodes to search!