
MidTerm: Wed. May.18 '05, 3pm - 4pm

Discrete Mathematics ICS 6A

Instructor: Max Welling

- *This exam is closed book*
- *Spend your time wisely: get a shot at each question.*
- *You don't have to simplify your answer using a calculator.*
- *Good Luck !*

1.(8pts) Sets and Logic

Let U (the universal set) be equal to all integers \mathbf{Z} . Define the sets A, B, C, D as follows:

$$\begin{aligned}A &= \{x|(x \in \mathbf{Z}) \wedge (x < 1)\} \\B &= \{x|(x \in \mathbf{Z}) \wedge (x > 9)\} \\C &= \{x|(x = 2k) \wedge (k \in \mathbf{Z})\} \\D &= \{x|(x \in \mathbf{Z}) \wedge (x > 12) \wedge (x < 5)\}\end{aligned}$$

a.(4pts) Determine the following sets (any set notation treated in class is acceptable in answering your questions):

(i) $B \cup C$, (ii) $(A \cup C) \cap C$, (iii) \bar{D} , (iv) $A - C$.

answer: (i) $\{x|(x \in \mathbf{Z}) \wedge ((x > 9) \vee (x = \text{even}))\}$

(ii) C

(iii) $U = \mathbf{Z}$

(iv) $\{x|(x \in \mathbf{Z}) \wedge (x < 1) \wedge (x = \text{odd})\}$

b.(1pt) What is the powerset of $\{\emptyset\}$?

answer: $\{\emptyset, \{\emptyset\}\}$

c.(1pt) Give the truth table for $(p \wedge q) \rightarrow q$, where p and q are two propositions.

answer: T,T,T,T (a tautology)

d.(1pt) Is the following proposition a tautology $(p \vee q) \rightarrow p$?

answer: NO (construct truth table and observe that it can be false)

e.(1pt) Determine the truth value of the following proposition: $((F \leftrightarrow F) \wedge (T \leftrightarrow T)) \rightarrow F$ (F=false, T=true).

answer: F

2.(5pts) Counting M&Ms

Geoff buys one bag of M&Ms in a store. The bag contains 5 colors of M&Ms, with 10 M&Ms per color. M&Ms of the same color are indistinguishable.

a.(1pt) Mary arrives and she likes to have 7 M&Ms. In how many ways can Geoff give Mary 7 M&Ms if the order does not matter?

answer: $C(5 + 7 - 1, 7)$

b.(1pt) In how many ways can Geoff give Mary 7 M&Ms if the order does matter?

answer: 5^7

Geoff decides to mark each M&M by making small scratches on them. From now on all 50 M&Ms are distinguishable.

d.(1pt) In how many ways can Geoff give 7 M&Ms to Mary of the order does not matter?

answer: $C(50, 7)$

d.(1pt) In how many ways can Geoff give 7 M&Ms to Mary of the order does matter?

answer: $P(50, 7)$

e.(1pt) Finally, Geoff decides to take all his M&Ms back and put them into 5 distinguishable boxes, 10 each. In how many ways can Geoff do that (all M&Ms are still distinguishable)?

answer: $50!/(10!)^5$

3.(7pts) **Probability**

A gambler tosses a coin 12 times. The coin is biased and has a probability of $p = 3/4$ to come up heads. The order in which he tosses the coin matters.

a.(1pt) How many possible sequences can the gambler toss?

answer: 2^{12}

b.(1pt) What is the probability of tossing exactly 12 heads?

answer: $(3/4)^{12}$

c.(1pt) What is the probability of first tossing 6 heads and then 6 tails?

answer: $(3/4)^6(1/4)^6$

d.(1pt) What is the probability of tossing a total of 6 heads (in any position)?

answer: $C(12, 6)(3/4)^6(1/4)^6$

e.(1pt) What is the probability that the first head arrives at toss number 8?

answer: $(1/4)^7(3/4)$

Let X be a random variable counting the total number of heads in 12 tosses.

f.(1pt) What is the expected value of X , i.e. $E[X]$?

answer: $12(3/4)$

g.(1pt) Compute the standard deviation of X , i.e. $STD(X)$.

answer: $\sqrt{12(3/4)(1/4)}$

4.(2pts) **Counting**

a.(1pt) How many ways are there to create new words from the word "IRVINE" by permuting the letters around?

answer: $6!/2!$

b.(1pt) How many new words can we create if a word must contain the block "NE"?

answer: $5!/2!$