Load balancing on a heterogeneous cluster.

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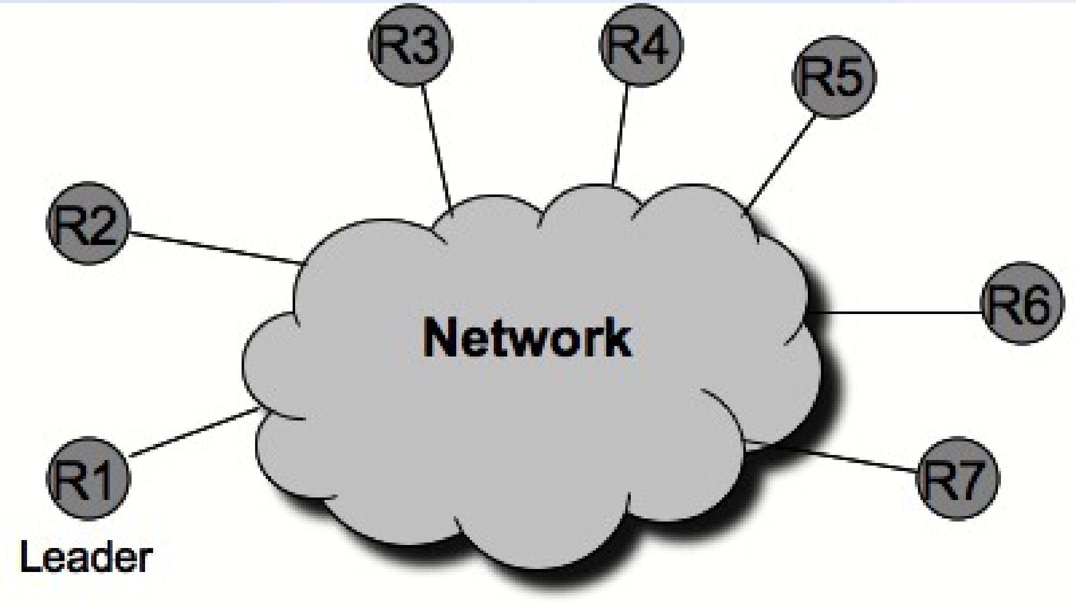
Dynamic Load-Balancing on Heterogeneous Clusters.

An extension of the PhD dissertation work of John Duselis, under Isaac Scherson.

Heterogeneous Cluster Connected Via Network



An Heterogeneous Cluster

Each resource *i* is represented by its Performance Vector (PV):

$$P_i = \begin{pmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_m \end{pmatrix}$$

- CPU
- Memory
- GPU
- HDD
- Bandwidth
- Et cetera

The tasks

A task T represents a computation to execute.

Define K distinct classes of tasks. Membership of a task to a class is determined by a classifier (not covered here).

A class k is defined by its resources requirements:

$$K_i = \begin{pmatrix} \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_m \end{pmatrix}$$

Execution Times

Thus, an approximation to the time taken by resource i to execute a task of class k is:

$$TaskExecutionTime = max_j \left(\frac{T_k[j]}{P_i[j]} \right) forall \ j = 1, \ldots, m$$

And an approximation to the time taken by resource i to execute all its assigned task:

$$ExecutionTime = \sum_k N_{i,k} \, max_j \left(\frac{T_k[j]}{P_i[j]} \right) foreach \ j = 1, \ldots, m$$

Where $N_{i,k}$ denotes the number of tasks of class k assigned to i.

Assignment of Tasks

N task are to be assigned. Each task belongs to a class k. Let:

I : Task is assigned to server i.

K: Task is of type k.

$x_{i,k}$ = Prob(I| K)

$$= \frac{Prob(I, K)}{Prob(K)}$$

$$Prob(I, K) = x_{i,k} Prob(K)$$

Note that the conditional probability of assigning N tasks of class k to a server i follows a multinomial probability

Expected Execution Times

The expected execution time for a server i is:

$$E\left[ExecutionTime\right]=E\left[\sum_k N_{i,k}\, max_j\left(\frac{T_k[j]}{P_i[j]}\right)\right]$$

$$=\sum_k E\left[N_{i,k}\right] max_j\left(\frac{T_k[j]}{P_i[j]}\right)$$

$$=\sum_k N\, x_{i,k}\, Prob(K=k)\, max_j\left(\frac{T_k[j]}{P_i[j]}\right)$$

Since $N_{i,k}$ is Multinomial, its expected value is (N Prob(I,K))

Total Execution Time

Assuming p.d.f of K is known, then:

$$Prob(K=k)\,max_j\left(\frac{T_k[j]}{P_i[j]}\right)=q_{i,k}$$

...which is a constant value.

Total system execution time is:

$$=max_i\left(\sum_k x_{i,k}\,Prob(K=k)\,max_j\left(\frac{T_k[j]}{P_i[j]}\right)\right)$$

$$=max_i\left(\sum_k x_{i,k}\,q_{i,k}\right)$$

= Time of last server to finish

Problem Formulation

$$X' = \begin{pmatrix} x_{1,1} & x_{2,1} & & x_{1,s} \\ \ldots & \ldots & \ldots & \ldots \\ x_{1,s} & x_{2,s} & & x_{s,k} \end{pmatrix}$$

$$X_i = the\ ith\ column\ vector\ of\ X'$$

$$Q = \begin{pmatrix} q_{1,1} & q_{2,1} & & q_{1,s} \\ \ldots & \ldots & \ldots & \ldots \\ q_{1,s} & q_{2,s} & & q_{s,k} \end{pmatrix}$$

$$Q_i = the\ ith\ column\ vector\ of\ Q$$

$$X = \begin{vmatrix} x_{1,1} \\ \ldots \\ x_{1,s} \\ x_{2,1} \\ \ldots \\ x_{2,s} \\ \ldots \\ x_{1,s} \\ \ldots \\ x_{k,s} \end{vmatrix}$$

Minimization Problem

$$argmin_X \, max_i \left(X_i C_i^T \right)$$

Subject to:

$$X_{i,k} \geq 0$$

$$J_{i,k} X_i = 1$$

Minimization Problem

$$argmin_X t$$

Subject to:

$$X_i C_i^T - t \leq 0$$

$$-X_{i,k} \leq 0$$

$$J_{i,k} X_i - 1 = 0$$

KKT Conditions

$$P_i C_i^T \leq t$$

$$P_{i,k} \geq 0$$

$$J_{i,k} P_i = 1$$

$$\lambda_i \geq 0$$

$$-\lambda_{i',k} P_{i,k} = 0$$

$$\lambda_i C_i^T - \lambda_{i',k} 1 + \nu J_{i,k} = 0$$

Accounting for Variance

Assume that:

$$Y_{i,k} := max_j \left(\frac{T_k[j]}{P_i[j]} \right)$$ be a random variable.

From previous problem we know:

$$ExecutionTime = \sum_k N_{i,k} max_j \left(\frac{T_k[j]}{P_i[j]} \right)$$

$$Var[ExecutionTime] = Var\left[\sum_k N_{i,k} Y_{i,k} \right]$$

Accounting for Variance

$$Var\left[ExecTime\right]=E\left[Y^2\right]\left(N^2p^2+Np(1-p)\right)+E\left[Y\right]^2\left(Np(1-p)-N^2p^2\right)$$

Is a quadratic convex function, let's denote this by:

$$V(i)$$

(Variance in execution time for sever i)

New Minimization Problem

$$argmin_X \, max_i \left(X_i C_i^T + s V(i) \right)$$

Subject to:

$$-X_{i,k} \leq 0$$

$$J_{i,k} X_i - 1 = 0$$

Where s is a parameter to be found using training set. Where $s \geq 0$

New Minimization Problem

$$argmin_X t$$

Subject to:

$$X_i C_i^T + s V(i) - t \leq 0$$

$$-X_{i,k} \leq 0$$

$$J_{i,k} X_i - 1 = 0$$

This is a Quadratically Constrained Quadratic Program (QCQP).