Math 291B/CS295 Assignment I Spring, 2011

READING: [FW] Chapters 1: Sections 1 - 4

EXERCISES: Hand in on Monday, April 3, 2011.

1. On a small island, 250 people have been exposed to an infectious disease. Consider an experiment which consists of determining the number N of people on the island who have the disease.

a) What is the sample space of the experiment?
b) Will this space be an equiprobable space?
c) If E is the event that $N \le 50$, what is E^c the complement of E?
d) If A us the event $N \ge 40$, what are A^c, $A \cup E$ and $A \cap E$?

2. Show $P(E_1 \cup E_2) \le P(E_1) + P(E_2)$ for any two events E_k, not necessarily elementary.

3. Show that the eigenvalues of M^T is the same as that of M and the eigenvector of the transition matrix M^T is the transpose of the eigenvector of M.

4. If $\mathbf{x}(0) = \mathbf{p}$ is a probability vector, prove that $\mathbf{x}(n) = M^n\mathbf{p}$ is also.

5. Prove that the product of two probability matrices is a probability matrix. In particular, any power of a probability matrix is a probability matrix.

6. If $M > O$, show $\mathbf{y} = M\mathbf{x} > \mathbf{0}$ for any probability vector $\mathbf{x}$. (A matrix $M > O$ means that all elements of M are positive. A vector $\mathbf{p} > \mathbf{0}$ means all components of the vector are positive.)

7. If M is the transition matrix of a regular Markov chain and $\mathbf{x}(0) = \mathbf{p}$ is a probability vector, show that $\mathbf{x}(n) = M^n\mathbf{p}$ is a positive probability vector for sufficiently large n.

8. Obtain $\mathbf{x}(n)$ of the IVP for the first order linear system of difference equations $\mathbf{x}(n+1) = M\mathbf{x}(n)$, with $\mathbf{x}(0) = \mathbf{p} = (p_1, 1 - p_1)^T$ and

$$M = \begin{bmatrix} p_d & p_q \\ 1 - p_d & 1 - p_q \end{bmatrix}.$$

9. Determine the eigen-pairs of the transition matrix for the social mobility problem,

$$M = \begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.3 & 0.8 & 0.2 \\ 0.1 & 0.1 & 0.7 \end{bmatrix},$$

and use it to solve the initial value problem with $\mathbf{x}(0) = \mathbf{c} = (c_1, c_2, c_3)^T$.

10. Prove that the Markov chain defined by the transition matrix A below is not a regular Markov Chain:

$$A = \begin{bmatrix} 1 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1 \end{bmatrix}.$$