Math 291B/CS295 | Assignment III | Spring, 2011

READING: [FW] Ch. 2: Sec. 1 – 7.
EXERCISES: Hand in Problems 1 – 8 to Professor Xie on Monday, April 18, 2011.

1. The difference operator D is defined on $\{x(n) \equiv x_n\}$ with $D[x(n)] = x(n+1) - x(n) = x_{n+1} - x_n$. Show

a) $D[\alpha x(n) \pm \beta y(n)] = \alpha D[x(n)] \pm \beta D[y(n)]$ (α and β are constants)

b) $D[x(n)y(n)] = D[x(n)]y(n) + x(n)D[y(n)] + D[x(n)]D[y(n)]$

c) $D[x(n)/y(n)] = \{D[x(n)]y(n) - x(n)D[y(n)]\}/[y(n)\{y(n) + D[y(n)]\}]$

2. With $x(0) = p$, solve the following IVP:

$$a)\quad x(n+1) = (n+1)x(n), \qquad b)\quad x(n+1) = (n+1)x(n) + (n+2).$$

3. Solve the IVP; $x(k+1) = \dfrac{x(k)}{\lambda + \alpha x(k)}$, $x(0) = p$ (λ and α are constants).

4. Solve the IVP: $x(n+1) - x(n) = \dfrac{2}{x(n+1) + x(n)}$, $x(0) = p$.

5. Solve the IVP: $x_{n+1}^2 - 3x_{n+1}x_n + 2x_n^2 = 0$, $x(0) = p$..

6. Solve the IVP: $x(n+1) = \sqrt{[x(n)]^3[x(n-1)]^2}$, $x(0) = p$, $x(1) = q$

7. Suppose in the two-allele Mendelian population model, only a fraction λ of the pure recessive genotype individuals does not participate in reproduction. Formulate a first order difference equation for the gene frequency q_n (with $p_n = 1 - q_n$).

8. The following questions pertain to the inbreeding model discussed during the April 7 lecture.

a) What is the initial probability distribution for the six possible states if the two parents are chosen at random for the normal populations? s

b) Formulate the transition matrix for the inbreeding model discussed in class.

c) Obtain the absorption matrix for this Markov chain.