Reading: [FW] Ch. 3


1. Prove Proposition 7 and Corollary 6 of Chapter 3.

2. Prove Propositions 9 and 10 (on Gaussian distribution) of Chapter 3.

3. Prove Propositions 11 and 12 along with Corollary 7 (on Poisson distribution) of Chapter 3.

4. A random variable is Rayleigh distributed with real-valued parameter \( \alpha > 0 \) if it is distributed by the density function

\[
p(x) = \begin{cases} 
\frac{x}{\alpha^2} e^{-\frac{x^2}{\alpha^2}} & (x \geq 0) \\
0 & (x < 0)
\end{cases}
\]

Determine \( P(X \leq z) \), \( E[X] \) and \( \text{Var}[X] \) for this density function.

5. A random variable is uniformly distributed in the interval \((a, b)\) if its density function is given by

\[
p(x) = \begin{cases} 
\frac{1}{b-a} & (a < x < b) \\
0 & (\text{otherwise})
\end{cases}
\]

Determine \( P(X \leq z) \), \( E[X] \) and \( \text{Var}[X] \) for this density function.

6. A random variable is exponentially distributed with real-valued parameter \( \lambda > 0 \) if its density function is given by

\[
p(x) = \begin{cases} 
\lambda e^{-\lambda x} & (x \geq 0) \\
0 & (x < 0)
\end{cases}
\]

Determine \( P(X \leq z) \), \( E[X] \) and \( \text{Var}[X] \) for this density function.

7. A random variable has a binomial distribution with real-valued parameter \( p \), \( 0 < p < 1 \), if its density function is given by

\[
p(x) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} \delta(x-k), \\
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

where \( \delta(z) \) is the Dirac delta function. Determine \( P(X \leq z) \), \( E[X] \) and \( \text{Var}[X] \) for \( p(x) \).

8. (to be added)