Math 291B/CS295 Assignment V Spring, 2011

READING: [FW] Ch. 3 and Ch. 4
EXERCISES: Hand in Problems 1 – 7 on Monday, May 10, 2011.

1. Do Example 14 of Chapter 3 of course notes.

2. Do Example 15 of Chapter 3 of course notes.

3. Prove Propositions 15 of Chapter 3.

4. Prove Proposition 16 of Chapter 4.

5. Prove Proposition 17 of Chapter 4.

6. Prove Proposition 18 of Chapter 4.

7. A random variable is *Cauchy distributed if it has a density function given by*

$$p(x) = \frac{a}{\pi(x^2 + a^2)} \qquad (a > 0).$$

Determine $P(X \le z)$, E[X] and Var[X] for this density function.

8. A random variable is *geometrically distributed* with success probability $p > 0$ if its density function is given by

$$p(x) = \sum_{k=0}^{\infty} \overline{p}\,\overline{q}^k \delta(x-k) \qquad (\overline{p} + \overline{q} = 1).$$

Show $E[X] = \dfrac{1}{p}$, $Var[X] = \dfrac{1-p}{p^2}$, $P(X = m) = p(1-p)^{m-1}$. (Note that $P(X = m)$ is the probability distribution of m Bernoulli trials needed to get one success.

9. Let $\{X_1, X_2, \ldots, X_n\}$ be n *i.i.d.* random variables with common mean μ and variance σ^2. Suppose $X = \frac{1}{n}\sum_{i=1}^{n} X_i$. Prove $E[X] = \mu$ and $Var[X] = \dfrac{\sigma^2}{n}$.

10. Suppose X is a *Gaussian* random variable with $\mu = 100$ and $\sigma^2 = 15$. Use the Chebyshev inequality to determine a lower bound for $P(|X - \mu| \le 20)$. Compare this value with one computed by using $N(100,15)$ (and you may use Mathematica or MatLab to find $N(100,15)$).

11. Suppose that X and Y are *i.i.d.* and both are uniformly distributed on the interval (0, 1). Show that the distribution of their sum is triangular on (0, 2). The density function for a symmetrically positioned triangular distribution is given by

$$p(x) = \begin{cases} \dfrac{1}{a^2}(a - x) & (|x| \le a) \\ 0 & (|x| \ge a) \end{cases}$$

(Let $a = 1$ and $x = z$ -- 1 for z in [0, 2] for the form of the triangular density for Z).