Math 291B/CS295 Assignment VI Spring, 2011

READING: FW] Ch. 5, Sec. 1 – 4.
EXERCISES: Hand in Problems 2 – 10 on Tuesday, May 17, 2011. Problem 1 is (optional) for extra credit if you do it right and hand it in on May 17.

1. (Poisson increments) Suppose $X(t)$ is Poisson distributed and $Y(t) = [X(t+\varepsilon) - X(t)]/\varepsilon$. $Y(t) = k/\varepsilon$ where k is the number of points in the interval $(t, t+\varepsilon)$ with $P(Y(t) = k/\varepsilon) = e^{-\lambda\varepsilon}(\lambda\varepsilon)^k/k!$. Show

$$E[Y(t)] = \lambda, \qquad C_{YY}(t_1,t_2) = \begin{cases} \lambda^2 \\ \lambda^2 + \dfrac{\lambda}{\varepsilon} + \dfrac{\lambda}{\varepsilon^2}|t_1 - t_2| \end{cases}$$

2. Prove Propositions 20 of Chapter 5.

3. Prove Propositions 21 of Chapter 5.

4. Prove the conclusion for Part i) of Example 23 of Chapter 5.

5. Prove the conclusion for Part ii) of Example 23 of Chapter 5.

6. Prove Lemmas 9 and 10 of Chapter 5.

7. Do Example 24 of Chapter 5.

8. Suppose a fair coin is tossed every T seconds starting from $t = 0$. Let the second order stochastic process $X(t)$ be defined by

$$X(t) = \left\{\begin{array}{ll} 1 & \text{if a head turns up at the n}^{\text{th}} \text{ toss} \\ -1 & \text{if a tail turns up at the n}^{\text{th}} \text{ toss} \end{array}\right\}, \qquad (n-1)T < t < nT.$$

Draw a sample path and show

$$(i) \quad E[X(t)] = 0, \qquad (ii) \quad E[X^2(t)] = 1,$$

$$(iii) \quad C_{XX}(t_1,t_2) = E[X(t_1)X(t_2)] = \begin{cases} 1 & |t_1 - t_2| < T \\ 0 & (otherwise) \end{cases}$$

9. Consider a s.p. $Y(t) = X(t - S)$ where $X(\bullet)$ is the s.p. of Problem 8 and S is a uniformly distributed random variable in $[0, T]$ and independent of $X(\bullet)$. Draw a sample path (with a typical shift due to S) and show

$$(i) \quad E[Y(t)] = 0, \qquad (ii) \quad C_{YY}(t_1,t_2) = \begin{cases} 1 - \dfrac{|t_1 - t_2|}{T} & (|t_1 - t_2| < T) \\ 0 & (otherwise) \end{cases}$$

10. a) For a Wiener process, show that $E[B(t)] = 0$ and $E[B^2(t)] = Dt$ keeping in mind

$$p(b;t) = \frac{1}{\sqrt{2\pi Dt}} e^{-b^2/2Dt}.$$

b) For $t > s$, the number of heads from tosses between s to t is independent of the those from tosses between 0 and s. Show that $C_{BB}(t_1,t_2) = D\min[t_1,t_2]$.