ICS 6N Computational Linear Algebra

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Teaching staff

• TA
  – Zana Ghaderi <zghaderi@uci.edu>

• Readers
  – Wentao Zhu
  – Efthymia Karra-Taniskidou

• Three discussion sessions (MWF 12:00-12:50pm SBSG 241)
  – Discuss lecture material
  – Coding assignment
Textbook


However, you are only responsible for materials covered in lectures and discussion sessions.
Obtaining Assistance

• Lecture and homework will be available from course website

• Use Piazza for class discussion
  – [https://piazza.com/uci/winter2017/ics6n/home](https://piazza.com/uci/winter2017/ics6n/home)
  – The system is highly catered to getting you help fast and efficiently from classmates, the TA, and myself. Rather than emailing questions to the teaching staff, I encourage you to post your questions on Piazza.

• Email us for private questions
Grading criteria

- Homework (25%) (10)
- Lab assignments (15%) (4)
- Two quizzes (30%)
- Final (30%)

Please carefully read policies on course website regarding academic honesty!
Topics

• Solving systems of linear equations
• Vector space, basis and dimension
• Least squares solutions
• Orthogonalization by Gram-Schmidt
• Properties of determinant
• Eigenvalues and eigenvectors
• Symmetric matrices and positive definite matrices
• Applications
Three components

• Notations: math vs. Matlab
• Algebra approaches
• Geometric approaches
Lecture 1

DATA TYPES, VECTORS, MATRICES
SCALARS

- Scalar is a real number
  - Examples: 1, 2.3, -0.2, 1005.6, 3.14159, -100

- If \( x \) is a real number, we usually say \( x \in \mathbb{R} \)

- More specifically, ‘hello world’ is not a real number.

- In linear algebra, we primarily deal with continuous real numbers.

- We will get to know complex numbers at later part of the course.
VECTORS

Vectors in $\mathbb{R}^2$

• Each vector consists of two real numbers
  – An example of a vector with two entries is $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

  where $w_1$ and $w_2$ are any real numbers.

• The set of all vectors with 2 entries is denoted by $\mathbb{R}^2$ (read “r-two”).
VECTORS

• The $\mathbb{R}^2$ stands for the real numbers that appear as entries in the vector, and the exponent 2 indicates that each vector contains 2 entries.
• Two vectors in $\mathbb{R}^2$ are equal if and only if their corresponding entries are equal.
• Given two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$, their sum is the vector $\mathbf{u}+\mathbf{v}$ obtained by adding corresponding entries of $\mathbf{u}$ and $\mathbf{v}$.
• Given a vector $\mathbf{u}$ and a real number $c$, the scalar multiple of $\mathbf{u}$ by $c$ is the vector $c\mathbf{u}$ obtained by multiplying each entry in $\mathbf{u}$ by $c$. 
VECTOR EQUATIONS

Example 1: Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find $4u$, $(-3)v$, and $4u + (-3)v$. 
VECTORS EQUATIONS

Example 1: Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find $4u, (-3)v$, and $4u + (-3)v$.

Solution: $4u = \begin{bmatrix} 4 \\ -8 \end{bmatrix}$, $(-3)v = \begin{bmatrix} -6 \\ 15 \end{bmatrix}$ and

$4u + (-3)v = \begin{bmatrix} 4 \\ -8 \end{bmatrix} + \begin{bmatrix} -6 \\ 15 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$
GEOMETRIC DESCRIPTIONS OF $\mathbb{R}^2$

- Consider a rectangular coordinate system in the plane. Because each point in the plane is determined by an ordered pair of numbers, *we can identify a geometric point $(a, b)$ with the column vector* $\begin{bmatrix} a \\ b \end{bmatrix}$.

- So we may regard $\mathbb{R}^2$ as the set of all points in the plane.
PARALLELOGRAM RULE FOR ADDITION

• If \( \mathbf{u} \) and \( \mathbf{v} \) in \( \mathbb{R}^2 \) are represented as points in the plane, then \( \mathbf{u} + \mathbf{v} \) corresponds to the fourth vertex of the parallelogram whose other vertices are \( \mathbf{u}, \mathbf{0}, \) and \( \mathbf{v} \). See the figure below.
Multiplication by a scalar

\[ u = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \]

Display \( u, 2u, -0.5u \)
VECTORS IN $\mathbb{R}^3$ and $\mathbb{R}^n$

- Vectors in $\mathbb{R}^3$ are vectors with three entries.
- They are represented geometrically by points in a three-dimensional coordinate space, with arrows from the origin.
- If $n$ is a positive integer, $\mathbb{R}^n$ (read “r-n”) denotes the collection of all lists (or ordered $n$-tuples) of $n$ real numbers, usually written as,

$$ u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} $$