1. Two students $A$ and $B$ are both registered for a certain course. Assume that student $A$ attends $80\%$ of the time, student $B$ attends $60\%$ of the time, and the absences of the two students are independent.

(a) What is the probability that at least one of the two students will be in class on a given day?
(b) If at least one of the two students is in class on a given day, what is the probability that $A$ is in class that day?

2. Suppose that $A$, $B$, and $C$ are three independent events such that $P(A) = 1/4$, $P(B) = 1/3$, and $P(C) = 1/2$.

(a) Determine the probability that none of these three events will occur.
(b) Determine the probability that exactly one of these three events will occur.

3. Let $A$ and $B$ be events for which $0 < P(A) < 1$ and $0 < P(B) < 1$.

(a) Prove that if $A$ and $B$ are independent, then $A \cap B \neq \emptyset$.
(b) Prove that if $A \cap B = \emptyset$, then $A$ and $B$ are not independent.

4. Assume that events $A$ and $B$ are independent given $D$. Show that

\[ P((A \cup B)|D) = P(A|D) + P(B|D) - P(A|D)P(B|D). \]