1. Evaluate $\lim_{n \to \infty} \left( 1 - \frac{a}{n} \right)^2$ where $a \in \mathbb{R}$.

2. Let $L(\mu) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\left( \frac{A - \mu}{2} \right)^2 \right]$ where $A$ is a constant. Derive $L'(\mu) = \frac{\partial}{\partial \mu} L(\mu)$.

3. Consider the function $f(x)$ such that
   
   $$f(x) = \begin{cases} 
   \frac{1}{\lambda} \exp(-\frac{x}{\lambda}), & x > 0 \\
   0, & x \leq 0
   \end{cases}$$

   where $\lambda$ is some positive-valued constant.

   (a) Evaluate $\int_{-\infty}^{\infty} f(x) dx$.

   (b) Find an expression for $\int_{-\infty}^{\infty} x f(x) dx$ in terms of $\lambda$.

   (c) Define $M(t) = \int_{-\infty}^{\infty} \exp(tx) f(x) dx$. Find an expression for $M(t)$ in terms of $t$ and $\lambda$.

   Find the range of values of $t$ under which $M(t)$ exists.

4. Recall the binomial theorem: For any $a, b \in \mathbb{R}$ and $N$ a positive-valued integer
   
   $$\sum_{n=0}^{N} \binom{N}{n} a^n b^{N-n} = (a + b)^N.$$ 

   Evaluate $\sum_{n=0}^{N} \binom{N}{n} \left( \frac{2}{7} \right)^n \left( \frac{5}{7} \right)^{N-n}$.

5. Find the Taylor series expansion (up to the second order only) for each of the following functions:

   (a) $f(x) = \exp(x)$

   (b) $f(x) = \ln(1 + x)$ where $x > -1$

6. Evaluate $\int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} (x - \mu)^2 \right] dx$. Hint: you may use the polar coordinate transformation.

7. Let $X_1$ and $X_2$ be independent random variables with means $\mu_1$ and $\mu_2$ respectively and variances $\sigma_1^2$ and $\sigma_2^2$ respectively.

   (a) For any constants $a_1$ and $a_2$, derive $\mathbb{E}(a_1 X_1 + a_2 X_2)$ and $\text{Var}(a_1 X_1 + a_2 X_2)$.

   (b) Find constants $c_1$ and $c_2$ so that $\frac{X_1 - c_1}{c_2}$ will have mean 0 and variance 1.
8. Let $X$ be a Gaussian random variable with mean $\mu$ and variance $\sigma^2$. Its probability density function (pdf) is given by

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right) \mathbb{I}_R(x)$$

(a) Let $M(t) = \int_{-\infty}^{\infty} \exp(tx)f(x|\mu, \sigma^2)dx$. Show that $M(t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2)$.

(b) Let $Y = (X-\mu)^2$. Derive $\mathbb{E} Y$ and $\text{Var} Y$.

9. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed as Poisson random variables with mean $\lambda$. Define $\overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_n$.

(a) Suppose that $n = 5$ and $\lambda = 2$. Find $\mathbb{P}(\overline{X}_n > 3)$. Express your final answer in terms of $\lambda$.

(b) Suppose that $\lambda = 2$. Use the CLT to approximate the probability $\mathbb{P}(\sum_{i=1}^{100} X_i < 300)$. 