1. (25) (Based on question 6.1 in Russel and Norvig) Consider a Tic-Tac-Toe game. Let $X_n$ be the number of rows, columns or diagonals containing exactly $n$ X’s and no O’s. Similarly, let $O_n$ be the number of rows, columns or diagonals containing exactly $n$ O’s and no X’s. We propose a utility function which assigns $+1$ to any position with $X_3 = 1$ (i.e. winning position) and assigns $-1$ to any position with $O_3 = 1$ (i.e. loosing position). The linear evaluation function we suggest is

$$3X_2 + X_1 - (3O_2 + O_1).$$

(a) How many states (i.e. board positions) are there in a Tic-Tac-Toe game. Note that there are symmetric board positions.

(b) What is the depth of the complete game tree? Does the complete game tree contain all the board positions you counted in (a)? Does it contain additional board positions?

(c) Show the game tree down to depth 2, namely starting from an empty board to all position in which there is one X and one O on the board.

(d) Mark on your tree the evaluation of the positions at level 2 (i.e. the value of the utility function at each position). Thereafter, mark on your tree the min-max values.

(e) Apply alpha-beta search on your tree, and mark the pruned subtrees when traversing from left to right, from right to left, and in an optimal order.

2. (10) Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player’s point of view. Assume that the first player is the maximizing player.

(a) What move should the first player choose?
Figure 1: A Game tree

(b) What nodes would not need to be examined using the alpha-beta algorithm—assuming that nodes are examined in left-to-right order?

3. (5) Consider a complete tree for a certain six-piece position end-game in checkers (Chinook computes such trees). Assume leaves of this tree are labeled by $-1$ if the position is a losing position, $+1$ if it is a winning position and $0$ if it is a tie. Is algorithm alpha-beta guaranteed to force a win whenever it is possible?

4. (5) Most game-playing programs do not save search results from one move to the next. Instead, they usually start completely over whenever it is the machine’s turn to move. Why?

5. (10) (Based on question 6.7 in Russel and Norvig) Prove that with a positive linear transformation of leaf values (i.e., transforming a value $x$ to $ax + b$, where $a > 0$), the choice of move remains unchanged in a game tree.

6. (15) Consider Alpha-Beta search.

(a) (10) Using the game tree in Figure 1 apply the alpha-beta procedure from right to left and determine which nodes will be pruned. Indicate where cutoffs occur. Repeat the problem from left to right.

(b) (5) What property should the leaf values of such tree have so that pruning will be maximized (resp. minimized) when the tree above is traversed from left to right.