Trees with Convex Faces and Optimal Angles

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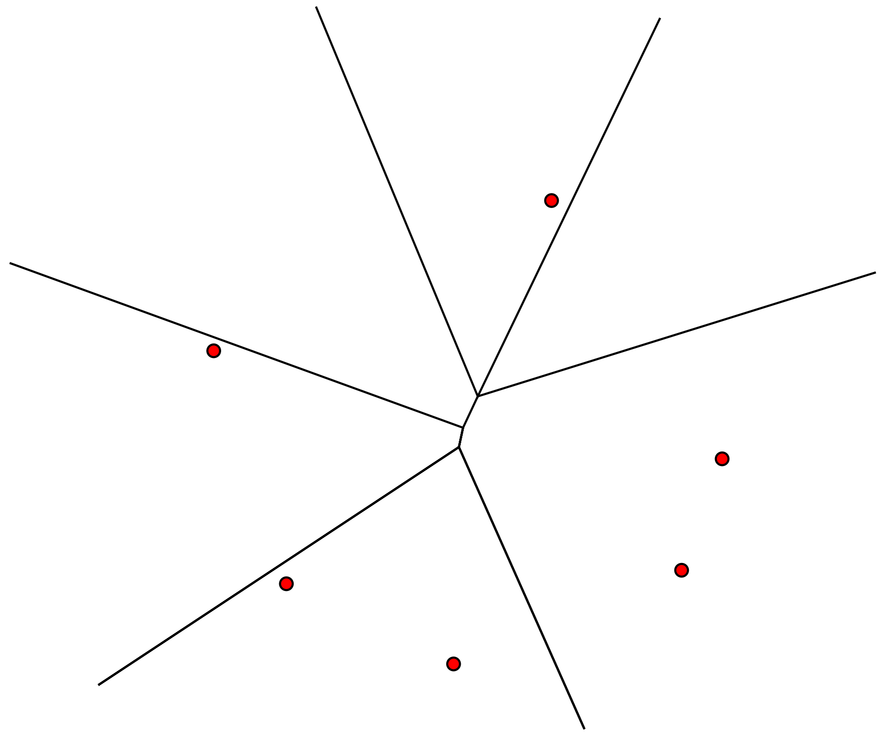
The Farthest Point Voronoi Diagram



Given a collection of point sites (shown as red dots)
partition the plane into regions within which one site is farther than all others

Forms an infinite convex cell for each convex hull vertex

The cell boundaries form a tree!

Known: Testing if a tree has a Voronoi representation

[Liotta and Meijer, CGTA 2003]

This paper:

Consider trees partitioning the plane into infinite convex cells
(without necessarily coming from a Voronoi diagram)

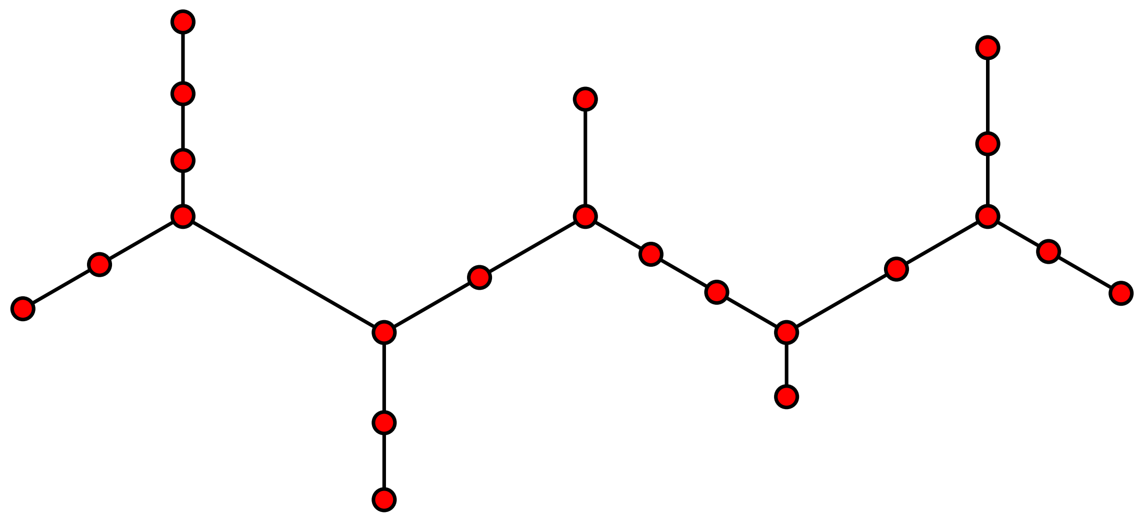
All trees have such a representation

What is the best representation for a given tree?

Our results:

Linear time algorithm for finding representation
with **optimal angular resolution** [Malitz, STOC 1992]

Rake = tree w/maximum degree three and with all degree-three nodes on a single path

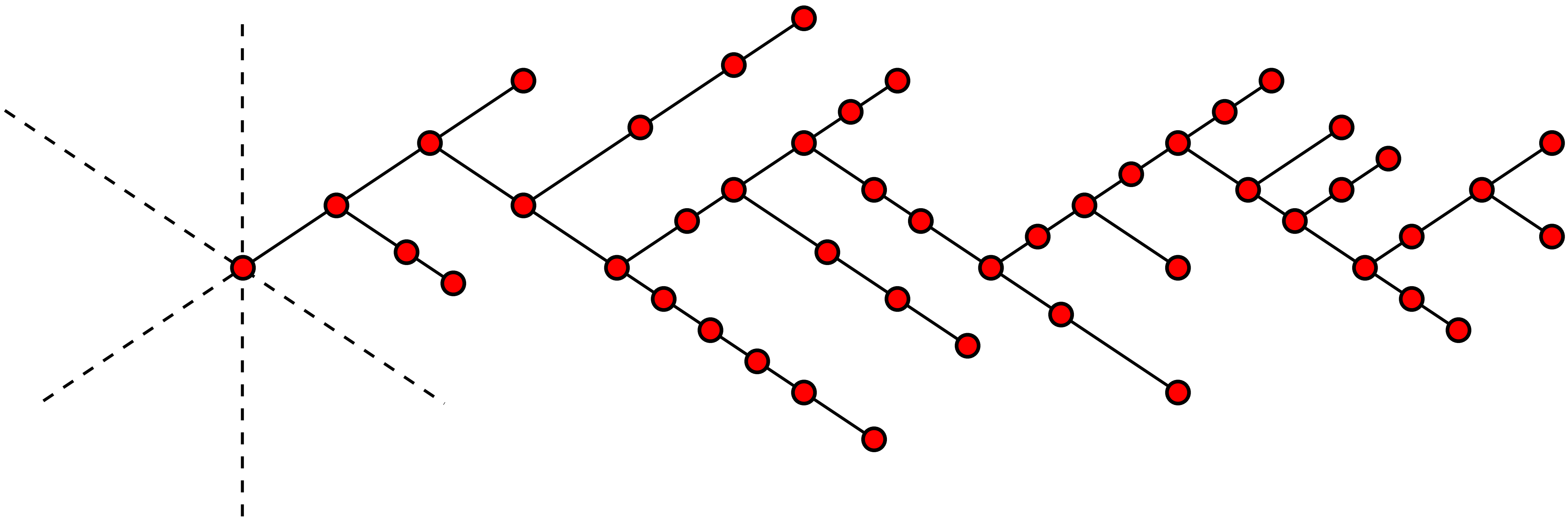


Theorem: **Any rake can be drawn with minimum angle 120 degrees**

(more complex formulae for plane-embedded rakes,
trees formed by connecting three rakes at a common node)

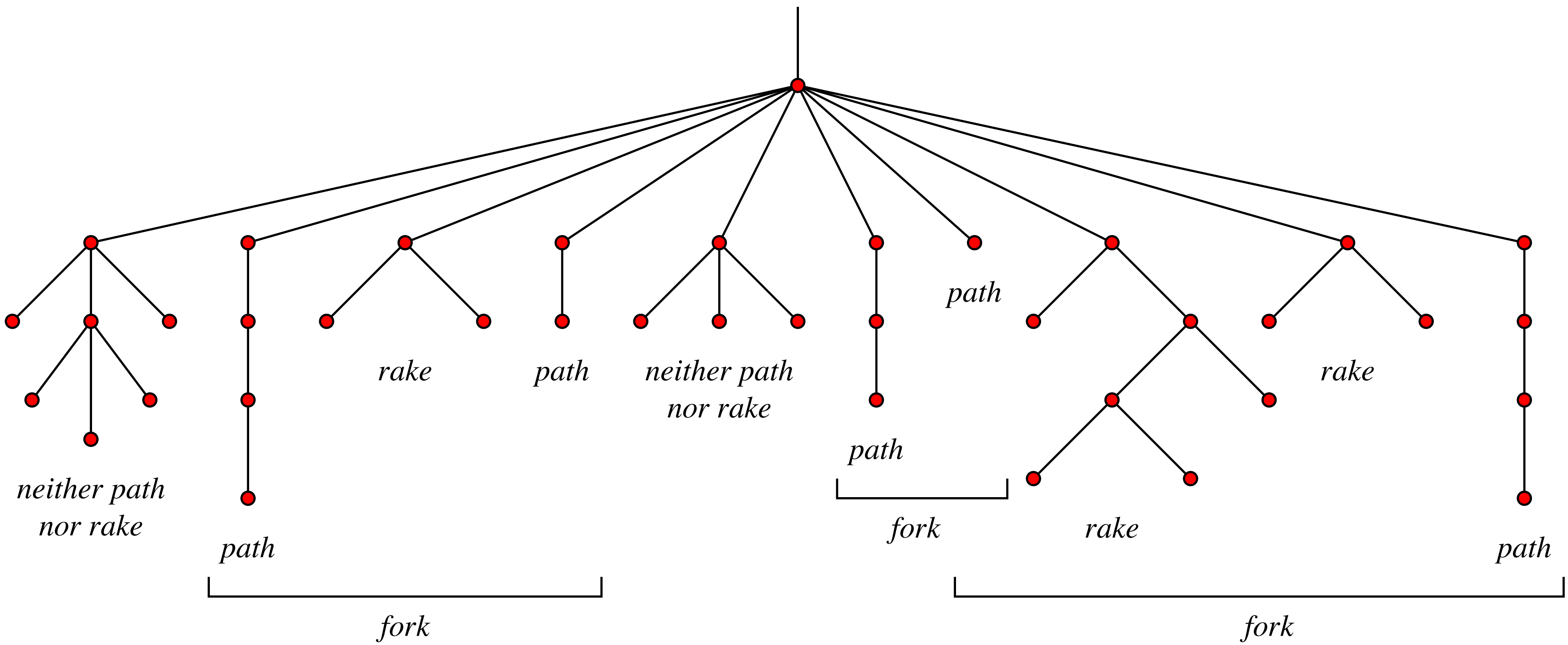
Rakes as part of larger trees

Can be drawn so that the range of angles used by rake edges
is the same as the minimum angle of the drawing



Fork

= consecutive subsequence of trees descending from children of some node forming the pattern: path – (0 or more rakes) – path



The Main Result

If we are given a tree that is neither a rake
nor three rakes joined at a common node

Then we can choose a root that is not part of any rake subtree

With this choice of root:

The sharpest angle in the optimal drawing of the tree
$= 2\pi$ / (# forks)

Algorithms

For plane-embedded trees:

Check for our two special cases (rakes or triple rakes)
Count forks
Assign slopes to edges in depth-first order

For unembedded trees:

Check for the two special cases
Embed the tree so as to minimize the number of forks
Apply the embedded algorithm

Both linear time!

How to set edge lengths once slopes are chosen?

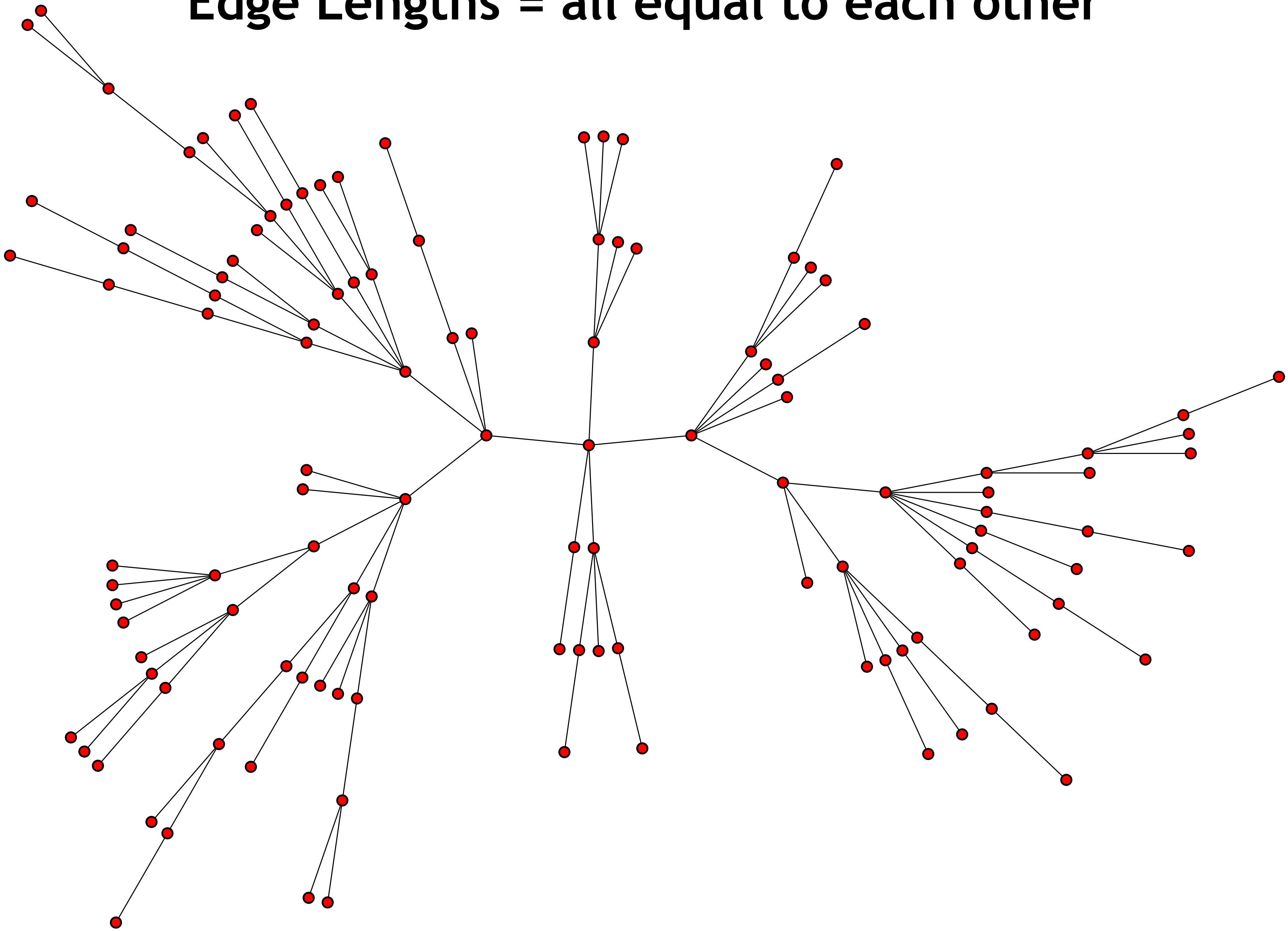
Any edge lengths will give a noncrossing drawing

So we're free to choose lengths either

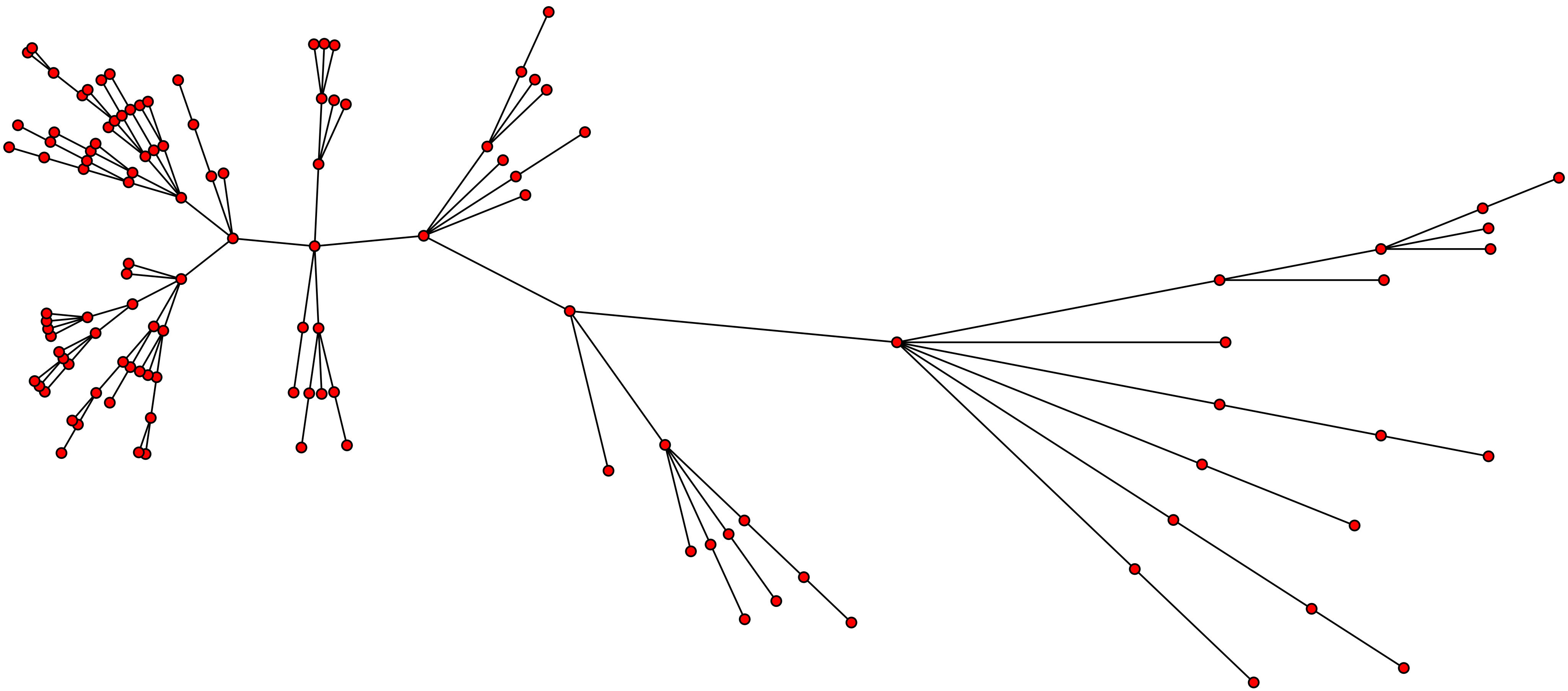
- to convey additional information about the tree
  (e.g. the location of and distance from the root)

- to enforce additional aesthetic criteria
  (e.g. vertex spacing)

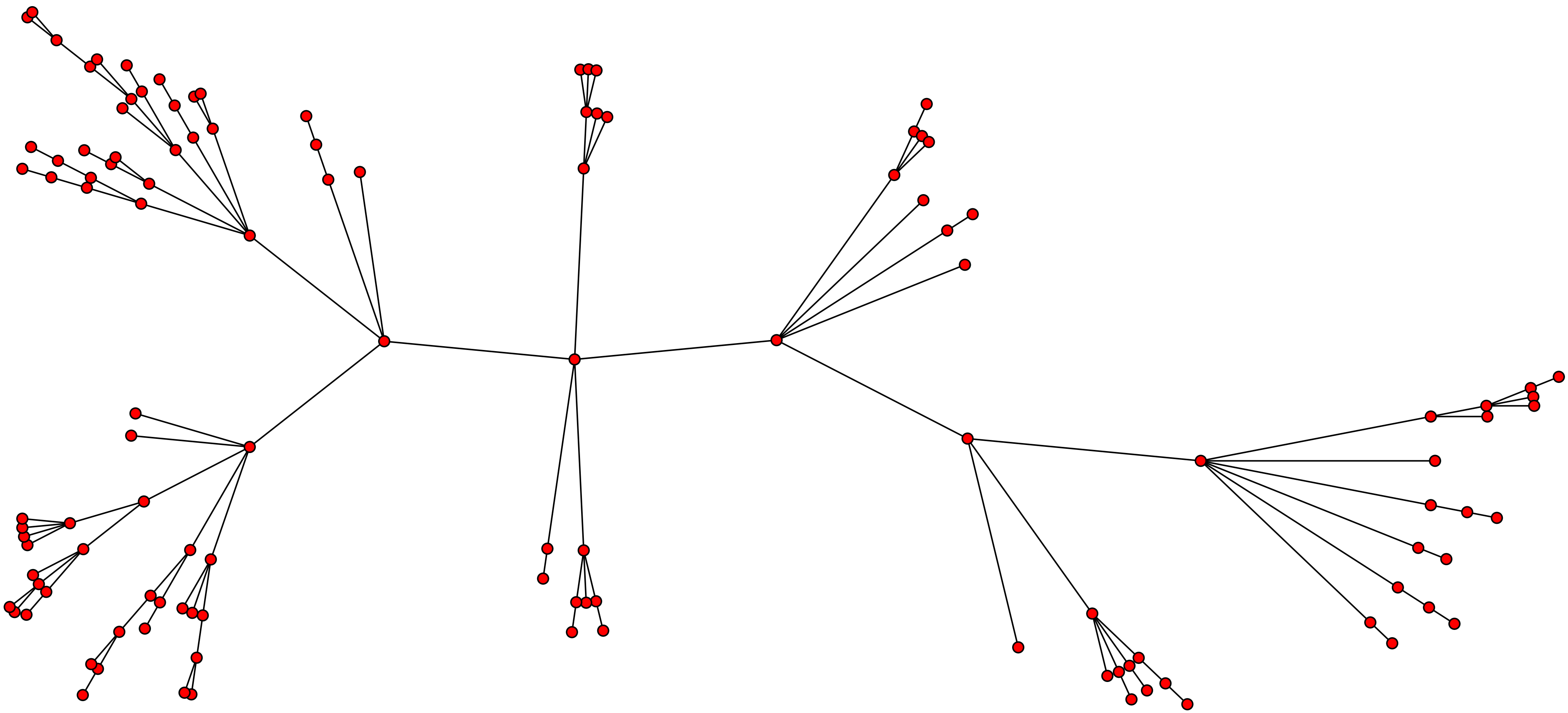
Edge Lengths = all equal to each other



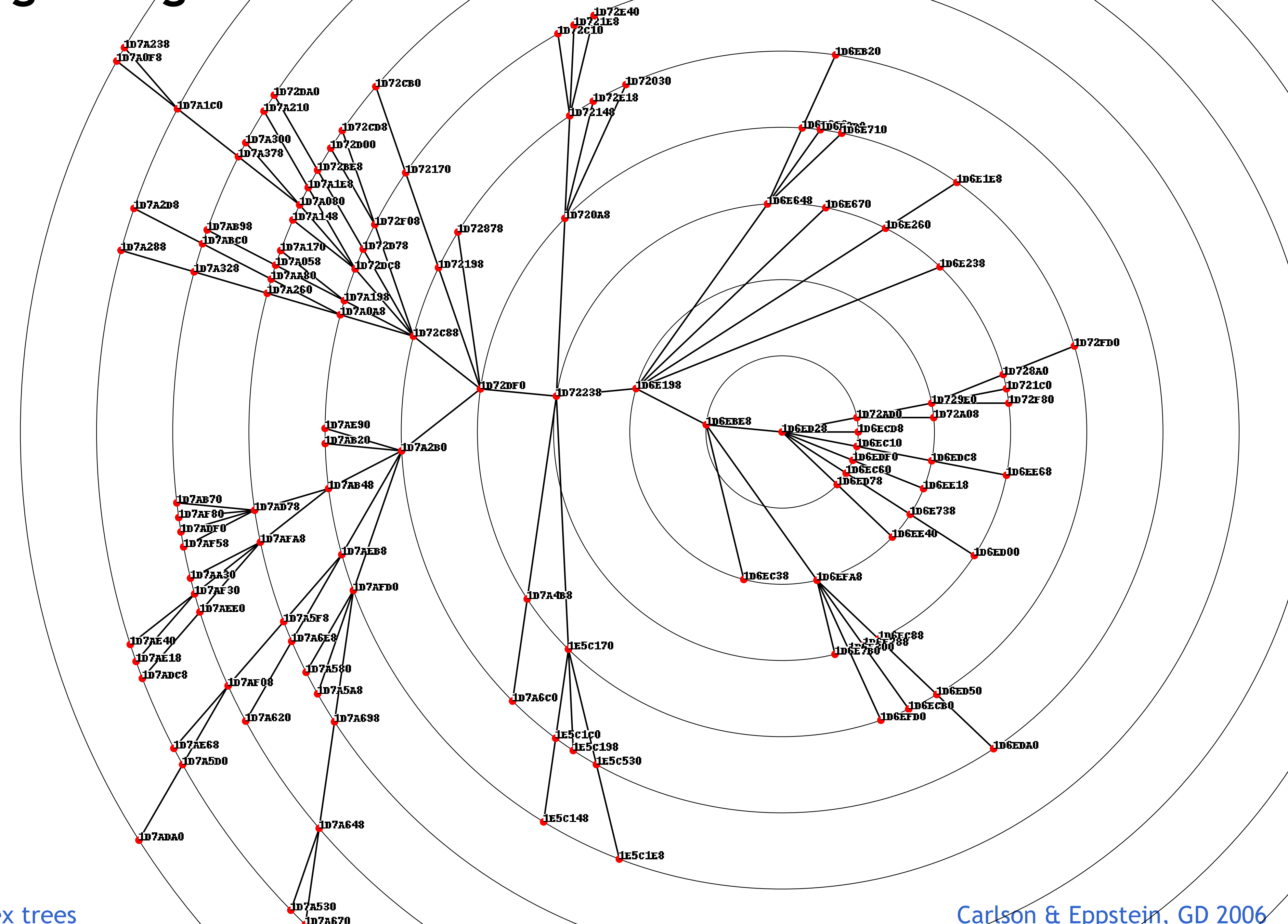
Edge Lengths = inverse of distance from root



Edge Lengths = sqrt(#descendants of top vertex)



Edge Lengths chosen so vertices lie on concentric circles



Convex trees

Carlson & Eppstein, GD 2006

Future Work

Dominance drawing for rooted trees?

Force all slopes to lie in a 180-degree arc
Same methods should extend straightforwardly

Other classes of graph?

Pseudotrees: tree plus single edge

Halin graphs: tree with cycle connecting leaves

Modified definitions of angular resolution?

Force paths to be drawn as zigzag lines
making vertices more visually apparent

Other optimality criteria?

Lexicographic vector of angles,
not just smallest angle