1. (30 points). Please answer the following short questions dealing with trees:

(a) Define \textit{external node} of a tree.

(b) What does it mean that a tree is an \textit{unordered} tree?

(c) If a tree has $n$ nodes in total, and $n \geq 3$, what is the maximum number of children that the root can have?
2. (30 points). Give a recursive, pseudo-code description of an algorithm for performing a postorder traversal of the subtree rooted at a node \( v \) in a binary tree \( T \). You may not use a loop in your description.
3. (30 points). Using the definition of Big-oh, show that each of the following is true.

(a) $3n^2 + 2n + 5$ is $O(n^2)$.

(b) $2n \log n + 2n + 10$ is $O(n \log n)$.

(c) $22 \log n + 10$ is $O(\log n)$. 
4. (30 points). This problem deals with linked lists.

(a) Describe a recursive algorithm for adding up the values stored at the nodes in a singly linked list \( L \) given a reference, \( f \), to its first node (the \texttt{next} pointer for the last node is null, and if \( f \) is null, then the list is empty). You may not use a loop for this question.

(b) Describe an algorithm for solving the above problem using an iterator \( x \) for the elements stored in \( L \). You must use a loop for this question.
5. (30 points). Consider the following algorithm, which takes as input an array $A$ of $n$ integers (indexed from 0 to $n - 1$):

$$i \leftarrow 1$$
$$p \leftarrow 1$$

while $i \leq n$ do {
$$p \leftarrow p \cdot A[i - 1]$$
$$i \leftarrow 2 \cdot i$$
}

Print “The product of the integers in $A$ is ” $p$

(a) Use the Big-oh notation to characterize the asymptotic running time of this algorithm, as a function of $n$ (you may assume that the array $A$ is already in memory; hence, you don’t have to count the time to read in $A$).

(b) For which values of $n$ is this a correct algorithm for multiplying all the integers in $A$?