1. (30 points). Short Answers.

(a) What is the worst-case running time for searching for an item \(x\) with key \(k\) in a hash table of size \(m\) that is storing \(n\) items?

(b) Suppose \(T\) is a hash table of capacity \(n\) that is storing \(m\) elements at random locations, such that \(m < n\), with collisions handled using chaining. Give a good upper bound on the probability that a new element added to \(T\) will collide with an existing element already in \(T\).

(c) What is the probability that an item \(y\) could be copied up to level \(i\) in a skip list?
2. (30 points). Heaps.

a. Draw a heap containing elements with keys from the set \{4, 6, 13, 17, 8, 16, 12, 9, 10\}.

b. Draw the result of adding an element with key 5 to your heap.
3. (30 points). Draw a picture that shows the final state of a hash table that uses a bucket array $B$, which has cells number 0 to 10, after we have inserted elements with keys from the set $\{4, 16, 3, 7, 5, 6, 2, 9, 12\}$ using the hash function

$$h(x) = (3x + 2) \mod 11.$$
4. (30 points). Describe an algorithm that uses an AVL tree to sort \( n \) comparable elements in \( O(n \log n) \) worst-case time.
5. (30 points). Suppose we are given an $n$-element sequence $S$ such that each element in $S$ represents a different vote in an election, where each vote is given as an integer representing the ID of the chosen candidate (vote IDs may be much larger than $n$). Assuming that there are at most $m$ candidates, design an efficient algorithm to see who wins the election $S$ represents, assuming the candidate with the most votes wins. What are the expected and worst-case running times of your method?