be a plane, and the columns of $A$ are linearly independent. By Theorem 12 in Section 1.9, the transformation $x \mapsto Ax$ is one-to-one.

24. $a = 1/\sqrt{5}$, $b = -2/\sqrt{5}$

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2. $A + 3B = \begin{bmatrix} 23 & -15 & 2 \\ 7 & -17 & -7 \end{bmatrix}$, $2C - 3E$ is not defined,

$DB = \begin{bmatrix} 26 & -35 & -12 \\ -3 & -11 & -13 \end{bmatrix}$, $EC$ is not defined.

4. $A - 5I_3 = \begin{bmatrix} 0 & -1 & 3 \\ -4 & -2 & -6 \\ -3 & 1 & -3 \end{bmatrix}$

$(5I_3)A = \begin{bmatrix} 25 & -5 & 15 \\ -20 & 15 & -30 \\ -15 & 5 & 10 \end{bmatrix}$

6. $a. \quad Ab_1 = \begin{bmatrix} -5 \\ 12 \\ 3 \end{bmatrix}$, $Ab_2 = \begin{bmatrix} 22 \\ -22 \\ -2 \end{bmatrix}$, $AB = \begin{bmatrix} -5 & 22 \\ 12 & -22 \\ 3 & -2 \end{bmatrix}$

$b. \quad AB = \begin{bmatrix} 4 \cdot 1 - 3 \cdot 3 & 4 \cdot 4 - 3 \cdot (-2) \\ -3 \cdot 1 + 5 \cdot 3 & -3 \cdot 4 + 5 \cdot (-2) \\ 0 \cdot 1 + 1 \cdot 3 & 0 \cdot 4 + 1 \cdot (-2) \end{bmatrix} = \begin{bmatrix} -5 & 22 \\ 12 & -22 \\ 3 & -2 \end{bmatrix}$

8. $B$ has 5 rows.

10. $AB = AC = \begin{bmatrix} -21 & -21 \\ 7 & 7 \end{bmatrix}$

12. By inspection of $A$, a suitable column for $B$ is any multiple of $(2, 1)$. For example: $B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$.

14. By definition, $UQ = [ Uq_1 \ldots Uq_4 ]$. From Example 6 in Section 1.8, the first column of $UQ$ lists the total costs for
15. a. False. See the definition of AB.
   b. False. The roles of A and B should be reversed in the second half of the statement. See the box after Example 3.
   c. True. See Theorem 2(b), read right to left.
   d. True. See Theorem 3(b), read right to left.
   e. False. The phase “in the same order” should be “in the reverse order.” See the box after Theorem 3.

16. a. True. See the box after Example 4.
   b. False. AB must be a 3 × 3 matrix, but the formula given here for AB implies that it is 3 × 1. The plus signs should be just spaces (between columns). This is a common mistake.
   c. True. Apply Theorem 3(d) to A² = AA.
   d. False. The left-to-right order of (ABC)T, is C T B T A T. The order cannot be changed, in general.
   e. True. This general statement follows from Theorem 3(b).

18. The third column of AB is also all zeros because
   \[ Ab_3 = A 0 = 0. \]

20. The first two columns of AB are Ab₁ and Ab₂. They are equal because b₁ and b₂ are equal.

22. If the columns of B are linearly dependent, then there exists a nonzero vector \( x \) such that \( Bx = 0 \). From this,
   \[ A(Bx) = A 0 = 0 \text{ and } (AB)x = 0 \text{ (by associativity)}. \]
   Since \( x \) is nonzero, the columns of AB must be linearly dependent.

24. Write \( I₃ = [e₁, e₂, e₃] \) and \( D = [d₁, d₂, d₃] \). By definition of AD, the equation AD = I₃ is equivalent to the three equations AD₁ = e₁, AD₂ = e₂, and AD₃ = e₃. Each of these equations has at least one solution because the columns of A span \( \mathbb{R}^3 \). (See Theorem 4 in Section 1.4.) Select one solution of each equation, and use them for the columns of D. Then AD = I₃.

26. Take any \( b \) in \( \mathbb{R}^m \). By hypothesis, \( ADb = Iₙb = b \). Rewrite this equation as \( A( Db ) = b \). Thus, the vector \( x = Db \) satisfies \( Ax = b \). This proves that the equation \( Ax = b \) has a solution for each \( b \) in \( \mathbb{R}^m \). By Theorem 4 in Section 1.4, A has a pivot position in each row. Since each pivot is in a different column, A must have at least as many columns as rows.

28. Since the inner product \( u^T v \) is a real number, it equals its transpose. That is, \( u^T v = (u^T v)^T = v^T (u^T)^T = v^T u \), by Theorem 3(d) regarding the transpose of a product of matrices and by Theorem 3(a). The outer product \( uv^T \) is an \( n \times n \) matrix. By Theorem 3, \((uv^T)^T = (v^T)^T u^T = vu^T\).

30. The (i, j)-entries of \( r(AB) \), \( r(A)B \), and \( A(rB) \) are all equal, because
   \[ r \sum_{k=1}^{n} a_{ik} b_{kj} = \sum_{k=1}^{n} (ra_{ik}) b_{kj} = \sum_{k=1}^{n} a_{ik} (rb_{kj}) \]

32. Let \( e_j \) and \( a_j \) denote the jth column of \( Iₙ \) and \( A \), respectively. By definition, the jth column of \( AIₙ \) is \( e_j \), which is simply \( a_j \), because \( e_j \) has 1 in the jth position and 0's elsewhere. Thus corresponding columns of \( AIₙ \) and \( A \) are equal. Hence \( AIₙ = A \).

34. By Theorem 3(d), \( (ABx)^T = x^T (AB) = x^T B^T A^T \).

36. [M] The answer will depend on the choice of matrix program. In MATLAB, the command \( \text{rand}(5,6) \) creates a 5 × 6 matrix with random entries uniformly distributed between 0 and 1. The command
   \[ \text{round}(19*(\text{rand}(4,4)-.5)) \]
creates a random 4 × 4 matrix with integer entries between −9 and 9. The same result is produced by the command \( \text{round} \text{int}(4,4) \) in the LayedAtA toolbox on the text website. On the TI-86 calculator, the corresponding command is \( \text{randM}(4,4) \).

38. The equation \( (A + I)(A - I) = A^2 - I \) is correct, however \( (A + B)(A - B) \neq A^2 - B^2 \) most of the time and hence will usually fail for random matrices.

40. The ones move out a super-diagonal for each consecutive power. Note \( S^3 = S^6 = 0 \).

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2. \[ \begin{bmatrix} -5 & 2 \\ 8 & -3 \end{bmatrix} \]

4. \[ \frac{1}{4} \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix} \text{ or } \begin{bmatrix} -3/2 & 1 \\ -1 & 1/2 \end{bmatrix} \]

6. \( x₁ = -5 \) and \( x₂ = \frac{26}{3} \)

8. Left-multiply each side of \( A = PBP^{-1} \) by \( P^{-1} \):
   \[ P^{-1} A = P^{-1} PBP^{-1}, \quad P^{-1} A = IBP^{-1}, \quad P^{-1} A = BP^{-1} \]

Then right-multiply each side of the result by \( P \):
   \[ P^{-1} AP = BP^{-1} P, \quad P^{-1} AP = BI, \quad P^{-1} AP = B \]

9. a. True, by the definition of invertible.
   b. False. See Theorem 6(b).
   c. False. If \( A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \), then \( ab - cd = 1 - 0 \neq 0 \), but Theorem 4 shows that this matrix is not invertible, because \( ad - bc = 0 \).
   d. True. This follows from Theorem 5, which also says that the solution of \( Ax = b \) is unique for each \( b \).
   e. True, by the box just before Example 6.

10. a. False. The last part of Theorem 7 is misstated here.