Product Rule: Finite Sets $A_1, \ldots, A_n$

$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|$

Lunch Special at a restaurant, Selection of Sandwich, Side, and a Drink.

Sandwiches = \{Burger, grilled chicken, grilled cheese\}
Sides = \{Fries, Fruit\}
Drinks = \{Soda, Diet Soda, OJ, Iced Tea\}

A meal selection specified by a tuple:

$([\text{Sandwich}], [\text{Side}], [\text{Drink}])$

For example: (Burger, fries, soda).

Total # of distinct choices for lunch special:

$|\text{Sandwiches} \times \text{Sides} \times \text{Drinks}| = 3 \times 2 \times 4 = 24$

Process of making a selection:

Multiply # selections at each point.

*Product rule works when the # choices at each point is independent of the choices made so far.*
A school has 5 4th-grade classes. The Student council consists of one representative chosen from each class.

\[ C_i = \text{Set of kids in the } i^{th} \text{ class. For } i = 1, 2, 3, 4, 5 \]

# Ways to select Student Council:

\[ |C_1| \cdot |C_2| \cdot |C_3| \cdot |C_4| \cdot |C_5| \]

Athlete training for a triathlon makes exercise schedule for the week. For each of the 7 days, she can run, swim or bike. How many different schedules are possible?

\[
\begin{array}{cccccc}
S & R & R & B & B & S & S \\
M & T & W & Th & F & Sa & Su
\end{array}
\]

3 \cdot 3 \cdot 3 \cdot \ldots \cdot 3 = 3^7
Counting strings of length $n$.

Set of symbols = $\Sigma$.

How many strings of length $n$ w/ symbol set $\Sigma$?

$|\Sigma^n| = |\Sigma|^n$  # binary strings of length 15

$= 2^{15}$

Specify each string as a selection process:
(binary strings of length 3).

$X \in \{0, 1\}^4$

$0\times \ 0\times \ 0\times \ 0$

$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
Sum Rule: If \( A_1, \ldots, A_n \) are pairwise disjoint (if \( i \neq j \Rightarrow A_i \cap A_j = \emptyset \)) then
\[
|A_1 \cup A_2 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|.
\]

Sandwiches = \{Burger, Grilled chicken, Billed cheese\}.
Sides = \{Fries, Fruit\}.
Hot Drinks = \{Coffee, tea, cocoa\}.
Cold Drinks = \{Water, soda\}.

Lunch consists of a Sandwich, a Side, a Drink \( \Rightarrow \) Hot Drinks \( \cap \) Cold Drinks = \( \emptyset \)

Drinks = Hot Drinks \( \cup \) Cold Drinks
\[
|\text{Drinks}| = |\text{Hot Drinks}| + |\text{Cold Drinks}|
\]
\[
= 2 + 3 = 5
\]

# Lunch Selections =
\[
3 \cdot 2 \cdot (2 + 3) = 3 \cdot 2 \cdot 5 = 30.
\]

\[
D = 30, 1, 2, 3, 4, 5, 6, 7, 8, 9, 5
\]
\[
I = \{A, B, C, D, \ldots, Y, Z\}.
\]

System passwords: String of digits or letters of length 7.

\# passwords: \[
\frac{36 \cdot 36 \cdot 36}{4} = (36)^7
\]
- length 7, must start w/ a letter:
  \[ 26 \cdot (36)^5 \]

- length 6, 7, or 8:
  \[ (36)^6 + (36)^7 + (36)^8 \]
Bijection Rule:

If $A \subset B$ are finite sets and there is a bijection between $A \subset B$ (f: $A \rightarrow B$ when f is a bijection) then $|A| = |B|$.  

Let $S$ be a finite set and $|S| = n$.

$S = \{x_1, x_2, \ldots, x_n\}$.

$|P(S)| = 2^n$  

$\Rightarrow$ Power set of $S$: Set of all subsets.

$f : P(S) \rightarrow \mathbb{R}^{0,1^m}$.

$A \subset S \quad f(A) = n$-bit string.

$j^{th}$ bit is 0 if $x_j \notin S$  

$j^{th}$ bit is 1 if $x_j \in S$.

$S = \{x_1, x_2, x_3, x_4, x_5\}$.

\[
\begin{align*}
A &= \{x_1, x_5\} \quad f(A) = 10001 \\
A' &= \{x_2, x_3, x_5\} \quad f(A') = 01101
\end{align*}
\]

Why is $f$ a bijection?

Each subset of $S$ uniquely specifies an $n$-bit string.  
Each $n$-bit string uniquely specifies a subset of $S$.  

A = \Sigma x_1, x_2, x_3 \cdot x_8^{-2}

f^n(x) = 10101.

E = \text{Set of 5-bit strings with an even \# of 1's.}

10110 \in E

10111 \in E

00000 \in E

Bijection between \(f: 30, 13^4 \to E\).