For the questions below, you do not need to come up with a final numerical answer. You can and should leave your solution as a mathematical expression, including \( \binom{n}{k} \) or \( P(n, k) \) notation as appropriate.

1. A ternary string has characters from the set \{0, 1, 2\}. For example 122010 and 0011210 are examples of ternary strings.
   (a) How many ternary strings are there whose length is in the range 6 through 8?

   (b) How many ternary strings of length seven start with a 1 or a 2?

   (c) How many ternary strings are there of length 8 with exactly three 1's?

   (d) How many ternary strings are there of length seven whose characters sum to a number that is equivalent to 0 mod 3?

2. A grocery store offers a promotion in which five customers visiting the store on a particular day are each given a gift. No customer can receive more than one gift. 250 customers enter the store on the day of the promotional.
   (a) If the gifts are all identical, how many different ways are there to distribute the gifts?

   (b) If the gifts are all different from each other, how many different ways are there to distribute the gifts?

3. Eight kids line up for a photo.
   (a) How many ways are there for the kids to line up?

   (b) One of the kids, Bob, has two close friends in the group. How many ways are there to line up the kids so that Bob’s two good friends are on either side of him (one to his immediate left and the other to his immediate right).