1. (5 pts) NAME AND EMAIL ADDRESS: 

YOUR ID: 
ID TO RIGHT: 
ROW: 
NO. FROM RIGHT: 

2. (5 pts) Write down the definition of \( P(H|D) \) in terms of \( P(H) \), \( P(D) \), \( P(H \text{ and } D) \), and \( P(H \text{ or } D) \).

\[
P(H|D) = \frac{P(H \text{ and } D)}{P(D)}
\]

3. (5 pts) Write down the expression that results from applying Bayes’ Rule to \( P(H|D) \).

\[
P(H|D) = P(D|H) \frac{P(H)}{P(D)}
\]

4. (5 pts) Write down the definition of \( A \rightarrow B \) in terms of “ and ”, “ or ”, and “not ”.

\[
A \rightarrow B = ((\text{not } A) \text{ or } B)
\]

(Other equivalent logical operators are OK.)

5. (5 pts each, 30 pts total) Mark the following statements as T (= true) or F (= false).

a. _____ T _____ \( P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B) \)

b. _____ T _____ \( P(A \text{ and } B) = P(A|B)P(B) \)

c. _____ T _____ \( P(A \text{ and } B) = P(A)P(B) \) if and only if A and B are independent.

d. _____ F _____ \( P(A \text{ or } B) = P(A) + P(B) \) if and only if A and B are independent.

e. _____ F _____ \( P(A \text{ and } B) = P(A)P(B) \) if and only if A and B are disjoint (do not intersect, or do not occur together).

f. _____ T _____ \( P(A \text{ or } B) = P(A) + P(B) \) if and only if A and B are disjoint (do not intersect, or do not occur together).
6. (5 pts each, 40 pts total) Let \( PKF(x, y) \) mean “Person \( x \) Knows Fact \( y \)”. For purposes of this question only, you may assume that the first argument is a person and the second is a fact.

For each English sentence below, write the logic sentence that best expresses it. Use “\( \neg \)" to mean “not.” The first one is done for you.

a. Every person knows every fact.  \( \forall x \forall y \ PKF(x, y) \).

b. Every person knows at least one fact.  \( \forall x \exists y \ PKF(x, y) \).

c. There is a person who knows at least one fact.  \( \exists x \exists y \ PKF(x, y) \).

d. There is a person who knows every fact.  \( \exists x \forall y \ PKF(x, y) \).

e. No person knows every fact.  \( \neg \exists x \forall y \ PKF(x, y) \).

Equivalent:  \( \forall x \exists y \neg PKF(x, y) \).

f. There is a person who knows no fact.  \( \exists x \forall y \neg PKF(x, y) \).

g. No person knows any fact.  \( \forall x \forall y \neg PKF(x, y) \).

h. There is a fact that is known by every person.  \( \exists y \forall x \ PKF(x, y) \).

i. There is a fact that no person knows.  \( \exists y \neg \exists x \ PKF(x, y) \).

Equivalent:  \( \exists y \forall x \neg PKF(x, y) \).

7. (2 pts each, 10 pts total) Fill in each blank below with Y (= Yes) or N (= No) depending on whether the logic expression correctly expresses the English.

a.  \( \mathbf{N} \)  “All cats are mammals.”  \( \forall x \text{Cat}(x) \& \text{Mammal}(x) \)

“All cats are mammals.” is \( \forall x \text{Cat}(x) \Rightarrow \text{Mammal}(x) \).
\( \forall x \text{Cat}(x) \& \text{Mammal}(x) \) is “Everything is a cat and a mammal.”

b.  \( \mathbf{Y} \)  “Spot has a sister who is a cat.”  \( \exists x \text{Sister}(x, \text{Spot}) \& \text{Cat}(x) \)

c.  \( \mathbf{N} \)  “For every person, there is someone that that person likes.”  \( \exists x \forall y \text{Likes}(x, y) \)

“For every person, there is someone that that person likes.” is \( \forall x \exists y \text{Likes}(x, y) \).
\( \exists x \forall y \text{Likes}(x, y) \) is “There is someone who likes everyone.”

d.  \( \mathbf{N} \)  “There is someone who is liked by everyone.”  \( \forall x \exists y \text{Likes}(x, y) \)

“There is someone who is liked by everyone.” is \( \exists y \forall x \text{Likes}(x, y) \).
\( \forall x \exists y \text{Likes}(x, y) \) is “For every person, there is someone that that person likes.”

e.  \( \mathbf{Y} \)  “Everyone likes ice cream.”  \( \neg \exists x \neg \text{Likes}(x, \text{IceCream}) \)