Knowledge Representation using First-Order Logic (Part II)

Reading: Chapter 8, 9.1-9.2
First lecture slides read: 8.1-8.2
Second lecture slides read: 8.3-8.4
Third lecture slides read: Chapter 9.1-9.2
(lecture slides spread across two class sessions)

(Please read lecture topic material before and after each lecture on that topic)
Outline

- Review: Follows, Entails, Derives
  - Follows: “Is it the case?”
  - Entails: “Is it true?”
  - Derives: “Is it provable?”
- Semantics of FOL (FOPC)
- FOL can be TOO expressive, can offer TOO MANY choices
  - Likely confusion, especially for teams of Knowledge Engineers
  - Different team members can make different representation choices
  - E.g., represent “Ball43 is Red.” as:
    - a predicate (= verb)? E.g., “Red(Ball43)” ?
    - an object (= noun)? E.g., “Red = Color(Ball43)” ?
    - a property (= adjective)? E.g., “HasProperty(Ball43, Red)” ?
  - SOLUTION: An upon-agreed ontology that settles these questions
    - Ontology = what exists in the world & how it is represented
    - The Knowledge Engineering teams agrees upon an ontology BEFORE they begin encoding knowledge
FOL (or FOPC) Ontology:
What kind of things exist in the world?
What do we need to describe and reason about?
Objects --- with their relations, functions, predicates, properties, and general rules.
If KB is true in the real world, then any sentence $\alpha$ entailed by KB and any sentence $\alpha$ derived from KB by a sound inference procedure is also true in the real world.
Schematic Example: Follows, Entails, and Derives

**Inference**

"Mary is Sue’s sister and Amy is Sue’s daughter."

"An aunt is a sister of a parent."

**Derives**

"Mary is Amy’s aunt."

**Entails**

"An aunt is a sister of a parent."

**Follows**

"Mary is Sue’s sister and Amy is Sue’s daughter."

**World**

Mary  Sister  Sue  Daughter  Amy  Mary  Aunt  Amy
Review: Models (and in FOL, Interpretations)

- Models are formal worlds in which truth can be evaluated

- We say $m$ is a model of a sentence $\alpha$ if $\alpha$ is true in $m$

- $M(\alpha)$ is the set of all models of $\alpha$

- Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$
  - E.g. $KB, = \text{“Mary is Sue’s sister and Amy is Sue’s daughter.”}$
  - $\alpha = \text{“Mary is Amy’s aunt.”}$

- Think of $KB$ and $\alpha$ as constraints, and of models $m$ as possible states.
- $M(KB)$ are the solutions to $KB$ and $M(\alpha)$ the solutions to $\alpha$.
- Then, $KB \models \alpha$, i.e., $\models (KB \Rightarrow \alpha)$, when all solutions to $KB$ are also solutions to $\alpha$. 
• $KB =$ all possible wumpus-worlds consistent with the observations and the “physics” of the Wumpus world.
\( \alpha_1 = "[1,2] \text{ is safe}\), \(KB \models \alpha_1\), proved by model checking.

Every model that makes \(KB\) true also makes \(\alpha_1\) true.
Semantics: Worlds

- The world consists of objects that have properties.
  - There are relations and functions between these objects.
  - Objects in the world, individuals: people, houses, numbers, colors, baseball games, wars, centuries
    - Clock A, John, 7, the-house in the corner, Tel-Aviv, Ball43
  - Functions on individuals:
    - father-of, best friend, third inning of, one more than
  - Relations:
    - brother-of, bigger than, inside, part-of, has color, occurred after
  - Properties (a relation of arity 1):
    - red, round, bogus, prime, multistoried, beautiful
Semantics: Interpretation

• An interpretation of a sentence (wff) is an assignment that maps
  – Object constant symbols to objects in the world,
  – n-ary function symbols to n-ary functions in the world,
  – n-ary relation symbols to n-ary relations in the world

• Given an interpretation, an atomic sentence has the value “true” if it denotes a relation that holds for those individuals denoted in the terms. Otherwise it has the value “false.”
  – Example: Kinship world:
    • Symbols = Ann, Bill, Sue, Married, Parent, Child, Sibling, ...
    – World consists of individuals in relations:
      • Married(Ann,Bill) is false, Parent(Bill,Sue) is true, ...
Truth in first-order logic

- Sentences are true with respect to a model and an interpretation.
- Model contains objects (domain elements) and relations among them.
- Interpretation specifies referents for:
  - constant symbols → objects
  - predicate symbols → relations
  - function symbols → functional relations
- An atomic sentence $\text{predicate}(\text{term}_1, \ldots, \text{term}_n)$ is true iff the objects referred to by $\text{term}_1, \ldots, \text{term}_n$ are in the relation referred to by predicate.
Semantics: Models

- An interpretation satisfies a wff (sentence) if the wff has the value “true” under the interpretation.
- Model: A domain and an interpretation that satisfies a wff is a model of that wff
- Validity: Any wff that has the value “true” under all interpretations is valid
- Any wff that does not have a model is inconsistent or unsatisfiable
- If a wff w has a value true under all the models of a set of sentences KB then KB logically entails w
Models for FOL: Example
Syntactic Ambiguity

- FOPC provides many ways to represent the same thing.
  - E.g., “Ball-5 is red.”
    - HasColor(Ball-5, Red)
      - Ball-5 and Red are objects related by HasColor.
    - Red(Ball-5)
      - Red is a unary predicate applied to the Ball-5 object.
    - HasProperty(Ball-5, Color, Red)
      - Ball-5, Color, and Red are objects related by HasProperty.
    - ColorOf(Ball-5) = Red
      - Ball-5 and Red are objects, and ColorOf() is a function.
    - HasColor(Ball-5(), Red())
      - Ball-5() and Red() are functions of zero arguments that both return an object, which objects are related by HasColor.
    - ...

- This can GREATLY confuse a pattern-matching reasoner.
  - Especially if multiple people collaborate to build the KB, and they all have different representational conventions.
FOL (or FOPC) Ontology:
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Summary

• First-order logic:
  – Much more expressive than propositional logic
  – Allows objects and relations as semantic primitives
  – Universal and existential quantifiers
  – syntax: constants, functions, predicates, equality, quantifiers

• Knowledge engineering using FOL
  – Capturing domain knowledge in logical form

• Inference and reasoning in FOL
  – Next lecture

• Required Reading:
  – Chapter 8.1-8.4
  – Next lecture: 8.3-8.4
  – Next lecture: Chapter 9.1-9.2