Homework 1

Sections 1.1-1.3

*Please write your name and student ID number clearly at the top of your homework.*

If you have multiple pages, please make sure they are secured together.

You should turn in your homework to the drop box located on the 3rd floor of Bren Hall, around the corner from room 3013. Please be quiet around this area as there are offices nearby. Also, please do not ask the staff for a stapler or paper clips. (This gets pretty tiring with 400 students turning in their assignments.)

**Problem 1**
Indicate whether each statement below is a proposition. If the statement is a proposition, express the negation of the proposition in English or mathematical symbols.

- a. 9 is a prime number.
- b. \( \pi = \frac{22}{3} \).
- c. Have a nice day.
- d. This year is a leap year.
- e. A week has seven days.
- f. Do I owe you any money?

**Problem 2**
For each statement below, indicate whether it is true or false, assuming that the "or" in the sentence is the inclusive or. Then indicate whether the statement is true or false if the "or" is the exclusive or.

- a. February has 31 days or the number 5 is an integer.
- b. The number \( \pi \) is an integer or the sun revolves around the earth.
- c. 20 nickels is worth one dollar or a whale is a mammal.

**Problem 3**
The propositional variables, \( p, q, \) and \( s \) have the following truth assignments: \( p = T, q = F, r = T \). Give the truth value for the following compound propositions:

- a. \( p \lor q \)
- b. \( q \oplus r \)
- c. \( p \land r \)
- d. \( \neg r \)

**Problem 4**
Suppose that \( p, q, r, s, \) and \( t \) are all propositional variables.

- a. Describe in words when the expression \( p \lor q \lor r \lor s \lor t \) is true and when it is false.
- b. Describe in words when the expression \( p \land q \land r \land s \land t \) is true and when it is false.
Problem 5
Give the inverse, converse and contrapositive for each of the following statements:
   a. It it rains, I will bring my umbrella.
   b. If you get your homework done, you can go out tonight.
   c. If the patient takes the medication, then he will have some side effects.

Problem 6
The values for the propositional variables p, q, r, s, are determined as follows:
   o  p = true
   o  q = false
   o  r = false
   o  s = true

Determine the truth values for the following propositions:
   a. p ∨ ¬(q → s)
   b. (r → q) ∨ ¬p
   c. (s ↔ ¬q) ∨ (p ∧ ¬q)
   d. (r ∧ p) → s
   e. p → (q ∨ s)

Problem 7
Give a truth table for the following propositions:
   a. (p → q) ∧ (q → p)
   b. p ↔ (¬p ∨ q)
   c. (p → ¬r) ∨ (r ∧ q)

Problem 8
Which statements below evaluate to true?
   a. If 3 is a prime number then 4 is also a prime number.
   b. If January has 28 days, then 3 is a prime number.
   c. January has 28 days if and only if 3 is a prime number.
   d. 8 is an odd number if and only if 10 is a prime number.

Problem 9
Define the following propositions:
   ● j: you will be summoned to jury duty
   ● f: you have been convicted of a felony.
   ● v: you are registered to vote
Express each of the following English sentences with a logical expression:

a. If you have been convicted of a felony then you are not registered to vote.

b. You will be summoned to jury duty only if you have not been convicted of a felony and you are registered to vote.

c. Being registered to vote and never having been convicted of a felony are sufficient for you to be summoned for jury duty.

d. Being summoned for jury duty implies that you are registered to vote.

e. You will not be summoned for jury duty if you have been convicted of a felony or you are not registered to vote.

Problem 10
Consider the following pieces of identification a person might have in order to apply for a credit card:

- B: Applicant presents a birth certificate.
- D: Applicant presents a driver's license.
- M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions.

a. The applicant must present either a birth certificate, a driver's license or a marriage license.

b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

c. Applicant must present either a birth certificate or both a driver's license and a marriage license.

Problem 11
For each table, give a logical expression whose truth table is the same as the one given below:

a.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

b.

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Problem 12
Define the following propositions:
p: The weather is bad.
q: The trip is cancelled.
r: The trip is delayed.

Translate the following English sentence into logical expressions using the definitions above:
a. The weather is good.
b. The weather is bad, but the trip is not canceled.
c. The weather is bad and the trip is canceled or delayed.
d. The trip was neither cancelled nor delayed.
e. The weather was good and the trip was not delayed.