Homework 3  
Sections 1.7

Please write your name and student ID number clearly at the top of your homework.
If you have multiple pages, please make sure they are secured together.

You should turn in your homework to the drop box located on the 3rd floor of Bren Hall, around the corner from room 3013.

When asked to prove something using the laws of logic, each line should be labeled "Hypothesis", "Conclusion" or with one of the laws of logic from Section 1.4 or 1.7. You should also number your lines and when using a law of logic, you should indicate which lines in your proof you are using.

Problem 1
Use a truth table to show that the Resolution Rule is a valid argument:
\[
\begin{align*}
 p \lor q \\
 \neg p \lor r \\
 \therefore q \lor r
\end{align*}
\]

Problem 2
Show that the following argument is not valid:
\[
\begin{align*}
 p \lor q \\
 p \lor r \\
 \therefore q \lor r
\end{align*}
\]

Problem 3
Use the laws of logic to show that the following argument is valid:
\[
\begin{align*}
 p \rightarrow r \\
 \neg q \\
 \neg p \rightarrow q \\
 \therefore p \land r
\end{align*}
\]

Problem 4
Use the laws of logic to prove the conclusion from the hypotheses. Give propositions variable names in your proof. The hypotheses are:
- If I drive on the freeway, I will see the fire.
- I will either drive on the freeway or take surface streets. (You can use inclusive or here).
- I am not going to take surface streets.

Conclude that I will see the fire.
Problem 5
Use the laws of logic to prove the conclusion from the hypotheses. Give propositions and predicates variable names in your proof. Use the set of all students as the domain of discourse. The hypotheses are:

- Larry and Hubert are taking Boolean Logic.
- Any student who takes Boolean Logic can take Algorithms.

Conclude that Larry and Hubert can take Algorithms.

Problem 6
Use the laws of logic to prove the conclusion from the hypotheses. Give propositions and predicate variable names in your proof. Use the set of all people as the domain of discourse. The hypotheses are:

- Everyone who practices hard is a good musician.
- There is a member of the orchestra who practices hard.

Conclude that someone in the orchestra is a good musician.

Problem 7
Which of the following arguments are valid? Explain your reasoning. (If you argue that it's valid, you should provide a proof. If you argue that it's invalid, you should give an example (small domain and predicate values) in which all the hypotheses are true but the conclusion is false.)

- I have a student in my class who is getting an A. Therefore, John, a student in my class is getting an A.
- Every girl scouts who sells at least 50 boxes of cookies will get a prize. Suzy, a girl scout, got a prize. Therefore Suzy sold 50 boxes of cookies.

Problem 8
Use the laws of logic to show that \( \forall x (P(x) \land Q(x)) \) implies that \( \forall x Q(x) \land \forall x P(x) \).

Problem 9
Consider the argument:

\[
\exists x P(x) \\
\exists x Q(x) \\
\therefore \exists x (Q(x) \land P(x))
\]

What is the flaw in the following proof of the argument?

1. \( \exists x P(x) \)  
   Hypothesis
2. $P(c)$ for some $c$  Existential Instantiation
3. $\exists x \ Q(x)$  Hypothesis
4. $Q(c)$ for some $c$  Existential Instantiation
5. $P(c) \land Q(c)$  Conjunction
6. $\exists x \ (Q(x) \land P(x))$  Existential Generalization