Homework 9

Sections 5.6, 5.7, 6.4, 6.5

*Please write your name and student ID number clearly at the top of your homework.*

If you have multiple pages, please make sure they are secured together.

*Please put your homework on standard 8.5x11 paper. Smaller pieces of paper tend to get lost in the pile.*

You should turn in your homework to the drop box located on the 3rd floor of Bren Hall, around the corner from room 3013.

**Problem 1**
The relations below are all defined on a set of people. Which ones are equivalence relations. In the case that the relation is not an equivalence relation, give the condition that is violated.

a. \((x, y)\) is in the relation if person \(x\) is a first cousin of person \(y\) (i.e., a parent of person \(x\) is a sibling of a parent of person \(y\).

b. The relation is the transitive closure of the relation given in part a.

c. \((x, y)\) is in the relation if \(x\) and \(y\) have the same birth mother.

d. \((x, y)\) is in the relation of \(x\) knows \(y\)'s cell phone number.

**Problem 2**
Consider the equivalence relation on the set \(S = \{7, 2, 13, 44, 56, 34, 99, 31, 4, 17\}\) such that two numbers are equivalent if they have the same remainder when divided by 4. Give the partition of \(S\) defined by the equivalence relation.

**Problem 3**
Which 4-tuples are in the relation \(\{(a, b, c, d): a, b, c, d\text{ are positive integers and }abcd=4\}\)?

*Source: Sandy Irani.*

**Problem 4**
List the tuples in the relation \(\{(a, b, c): a, b, \text{ and } c\text{ are positive integers and }0 < a < b < c < 5\}\).

*Source: Sandy Irani.*

**Problem 5**
Consider the following small database whose records correspond to flights departing from an airport.

<table>
<thead>
<tr>
<th>Airline</th>
<th>Flight number</th>
<th>Gate</th>
<th>Destination</th>
<th>Departure time</th>
</tr>
</thead>
<tbody>
<tr>
<td>FlyRight</td>
<td>122</td>
<td>34</td>
<td>Detroit</td>
<td>08:10</td>
</tr>
<tr>
<td>JetGreen</td>
<td>221</td>
<td>22</td>
<td>Denver</td>
<td>08:17</td>
</tr>
<tr>
<td>JetGreen</td>
<td>122</td>
<td>33</td>
<td>Anchorage</td>
<td>08:22</td>
</tr>
<tr>
<td>JetGreen</td>
<td>323</td>
<td>34</td>
<td>Honolulu</td>
<td>08:30</td>
</tr>
<tr>
<td>FlyRight</td>
<td>199</td>
<td>13</td>
<td>Detroit</td>
<td>08:47</td>
</tr>
<tr>
<td>JetGreen</td>
<td>222</td>
<td>22</td>
<td>Denver</td>
<td>09:10</td>
</tr>
<tr>
<td>FlyRight</td>
<td>322</td>
<td>34</td>
<td>Detroit</td>
<td>09:44</td>
</tr>
</tbody>
</table>

a. Are the attributes **Airline** and **Destination** a key for this database?

b. Give examples of two separate sets of attributes that are keys for this database

c. Show the results of the operation `SELECT[Airline="JetGreen" ∧ Destination="Denver"]`

d. Show the results of the operation `PROJECT[Airline, Gate]`

e. Show the results of the operation `SELECT[Destination="Denver"]` followed by `PROJECT[Gate]`

f. What operation or operations should be performed if you wanted to know which airlines use gate 22?

g. What operation or operations should be performed if you wanted to know whether there are any flights to Detroit departing before 9:10? A flight is identified by the airline and the flight number.

Source: Sandy Irani.

**Problem 6**

The table below shows the database of course offerings for the Computer Science Department for the current academic year. It's actually just a portion of the course offerings, but for the purposes of this question, assume it lists all the courses.

<table>
<thead>
<tr>
<th>Course number</th>
<th>Course title</th>
<th>Instructor</th>
<th>Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS 111</td>
<td>Digital Image Processing</td>
<td>Majumder</td>
<td>Spring</td>
</tr>
<tr>
<td>CS 112</td>
<td>Computer Graphics</td>
<td>Majumder</td>
<td>Spring</td>
</tr>
<tr>
<td>CS 117</td>
<td>Project in Computer Vision</td>
<td>Fowlkes</td>
<td>Fall</td>
</tr>
<tr>
<td>CS 116</td>
<td>Computational Photography &amp; Vision</td>
<td>Fowlkes</td>
<td>Winter</td>
</tr>
<tr>
<td>CS 122A</td>
<td>Introduction to Data Management</td>
<td>Carey</td>
<td>Winter</td>
</tr>
<tr>
<td>CS 122A</td>
<td>Introduction to Data Management</td>
<td>Li</td>
<td>Winter</td>
</tr>
<tr>
<td>CS 122C</td>
<td>Principles of Data Management</td>
<td>Carey</td>
<td>Winter</td>
</tr>
</tbody>
</table>

a. Give a key for this database.

b. Show the results of the operation `SELECT[Course number = "CS122A" ∨ Course number = "CS122C"]`

c. Show the results of the operation `PROJECT[ Instructor ]`
d. Show the results of the operation `SELECT[Quarter=“Spring”]` followed by `Project[Course number]`. Express in English what question this combination of queries is asking.
e. What operation or operations should be performed if you wanted to know which quarters CS 122A is being offered?
f. What operation or operations should be performed if you wanted to know which courses are being taught by Prof. Majumder?

**Problem 7**
Below is the state diagram for a finite state machine. The start state is shown with a double circle.

Show the output of the finite state machine on the following inputs:

a. 000  
b. 010  
c. 111

**Problem 8**
Below is the state diagram for a finite state machine. The start state is shown with a double circle.

Show the output of the finite state machine on the following inputs:

a. 000  
b. 010  
c. 111

**Problem 9**
Below are the state diagrams for two finite state machines. The start state of each one is shown with a double circle.
Below are several descriptions of finite state machine. Which description correctly describes the output of A? Which description correctly describes the output of B?

- a. Each output bit of the finite state machine is the parity of all the input bits seen so far. Recall that the parity of a set of bits is 1 if the number of 1's is odd and 0 if the number of 1's is even.
- b. Each output bit of the finite state machine is the AND of all the input bits seen so far.
- c. Each output bit is the negation of the last input bit seen so far.
- d. Each output bit of the finite state machine is the OR of all the input bits seen so far?

**Problem 10**
Design a finite state machine whose input alphabet is \{0, 1\} that accepts a string if and only if there are no occurrences of "00" or "11".

**Problem 11**
Here is a description of a Turing Machine. The input alphabet is \{a, b\}. The state set is \{q_0, q_1, q_2, q_3, q_4, q_{acc}, q_{rej}\}. The transition function is given by the following table:

<table>
<thead>
<tr>
<th></th>
<th>q_0</th>
<th>q_1</th>
<th>q_2</th>
<th>q_3</th>
<th>q_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(q_1, *, R)</td>
<td>(q_1, a, R)</td>
<td>(q_2, a, R)</td>
<td>(q_{acc}, a, R)</td>
<td>(q_{rej}, a, R)</td>
</tr>
<tr>
<td>b</td>
<td>(q_2, *, R)</td>
<td>(q_1, b, R)</td>
<td>(q_2, b, R)</td>
<td>(q_{rej}, b, R)</td>
<td>(q_{acc}, b, R)</td>
</tr>
<tr>
<td>*</td>
<td>(q_{rej}, *, R)</td>
<td>(q_3, *, L)</td>
<td>(q_4, *, L)</td>
<td>(q_{rej}, *, R)</td>
<td>(q_{rej}, *, R)</td>
</tr>
</tbody>
</table>

- a. Draw the configuration of the Turing Machine after 3 steps on input "aabba".
- b. Simulate the Turing Machine on input "aabba". Does it accept?
- c. Simulate the Turing Machine on input "aabb". Does it accept?
- d. Describe in words the conditions under which the Turing Machine accepts.

**Problem 12**
Design a Turing Machine whose input alphabet is \{0, 1\} and accepts an input string if and only if the length of the string is even.