1. Consider the following graph:

(a) Construct the adjacency matrix and adjacency list for the above graph.
(b) Run dfs on the above graph and record the order vertices are visited. Assume neighbors are examined in alphabetical order.
(c) Run bfs on the above graph and record the order vertices are visited. Assume neighbors are examined in alphabetical order.
(d) Run Dijkstra’s algorithm on the above graph starting from vertex \( a \) and record the shortest distances to each node. Highlight or darken the edges that form the shortest paths.

2. Suppose a depth first search is run on a rooted binary tree where the neighbor ordering is left child and then right child. The order the DFS visits the nodes of the tree is the same as something else we’ve seen previously. What is it?

3. Dijkstra’s algorithm is guaranteed to compute the correct shortest path distances when the edge weights are non-negative. When run on graphs with negative edge weights, Dijkstra’s algorithm might return incorrect distances.
(a) Draw a graph and indicate a start vertex such that Dijkstra’s algorithm will succeed and return correct distances.

(b) Draw a graph and indicate a start vertex such that Dijkstra’s algorithm will fail and return incorrect distances.

4. The breadth first search we described in class outputs the vertices sorted by the number of edges between each vertex and the start vertex i.e. all the vertices one edge away from the start come before the vertices two edges away from the start and so on. Alter breadth first search to also compute the number of edges between each vertex and the start vertex.

Extra credit: The diameter of a graph is the largest distance between any two nodes. The small world phenomenon is the phenomenon that in certain graphs the diameter is very small. For example in the graph where the nodes are people and the edges are social relationships, the distance between any two people tends to be less than six giving rise to the term “six degrees of separation”. Describe an algorithm that runs in $O(n^2 \log n + nm)$ time to compute the diameter of a graph.