Discussion Section Week 1

- Intro
- Course Project Information
- Constraint Satisfaction Problems
  - Sudoku
  - Backtracking Search Example
  - Heuristics for guiding Search Example

Intro

- Teaching Assistant
  - Junkyu Lee (June Queue Lee)
  - Office Hour
    - Friday 11:00 AM ~ 12:00 PM
    - DBH 4099
- Reader
  - Minhaeng Lee (Min Heng Lee)
  - Office Hour
    - Thursday 2:00 PM ~ 3:00 PM
    - DBH 4219

Course Project Information

- **Fri., 15 Jan., 11:59pm: Project Team Formation**
- Sun., 24 Jan., 11:59pm: Project **Problem** Generator
- Sun., 31 Jan., 11:59pm: Project **Backtracking** Search
- Sun., 14 Feb., 11:59pm: Project **Forward Checking**
- Sun., 21 Feb., 11:59pm: Project **Arc Consistency**
- Sun., 28 Feb., 11:59pm: Project **MRV & DH Heuristic**
- Sun., 6 Mar.., 11:59pm: Project **LCV** Heuristic
- Sun., 13 Mar., 11:59pm: Final Project

You will lose 10% of your Project grade for every day or fraction thereof it is late

Course Project Information

- **Fri., 15 Jan., 11:59pm: Project Team Formation**
  - How Many Members ?
  - We will post a google doc next week on EEE message board

You Will Be Expected to Know

- Basic definitions (section 6.1)
  - What is a CSP?
- Backtracking search for CSPs (6.3)
- Variable ordering or selection (6.3.1)
  - Minimum Remaining Values (MRV) heuristic
  - Degree Heuristic (DH) (to unassigned variables)
- Value ordering or selection (6.3.1)
  - Least constraining value (LCV) heuristic

What is CSP?

- Task

- Model

What is CSP?

- Task/goal for solving CSP
  - Given a set of constraints,
    - Find a solution that satisfy all constraints
    - Find all solutions that satisfy all constraints
    - Count number of solutions
    - …

What is CSP?

- How to model/express CSP problems?
  - variable and its domain
  - constraints, relations, functions
    - **allowed** (partial) combinations of variable values

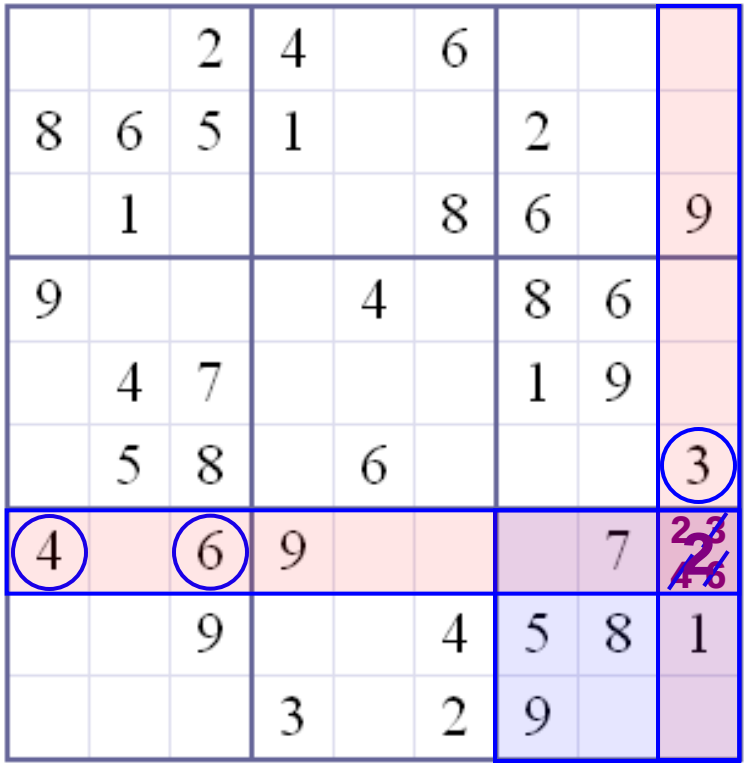
Constraint Satisfaction Problems

- What is a CSP?
  - Finite set of variables $X_1, X_2, \ldots, X_n$
  - Nonempty domain of possible values for each variable $D_1, D_2, \ldots, D_n$
  - Finite set of constraints $C_1, C_2, \ldots, C_m$
    - Each constraint C_i limits the values that variables can take,
    - e.g., $X_1 \neq X_2$
  - Each constraint C_i is a pair <scope, relation>
    - Scope = Tuple of variables that participate in the constraint.
    - Relation = List of allowed combinations of variable values.
      May be an explicit list of allowed combinations.
      May be an abstract relation allowing membership testing and listing.
- CSP benefits
  - Standard representation pattern
  - Generic goal and successor functions
  - Generic heuristics (no domain specific expertise).

Sudoku

Example: Sudoku (constraint propagation)

Constraint propagation

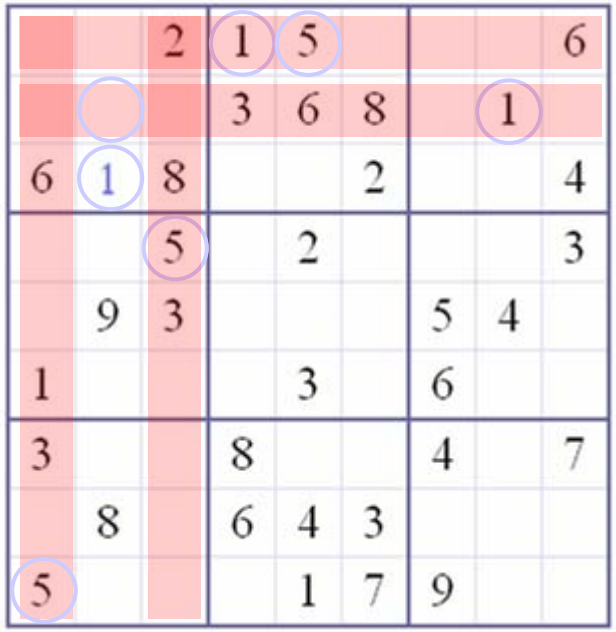


- **Variables**: 81 slots
- **Domains** = {1,2,3,4,5,6,7,8,9}
- **Constraints**:
  - 27 not-equal

Each row, column and major block must be alldifferent

"Well posed" if it has unique solution: 27 constraints

Sudoku (inference)



		2	1	5				6
			3	6	8		1	
6	1	8			2			4
		5		2				3
	9	3				5	4	
1				3		6		
3			8			4		7
	8		6	4	3			
5				1	7	9		

Each row, column and major block must be alldifferent

“Well posed” if it has unique solution

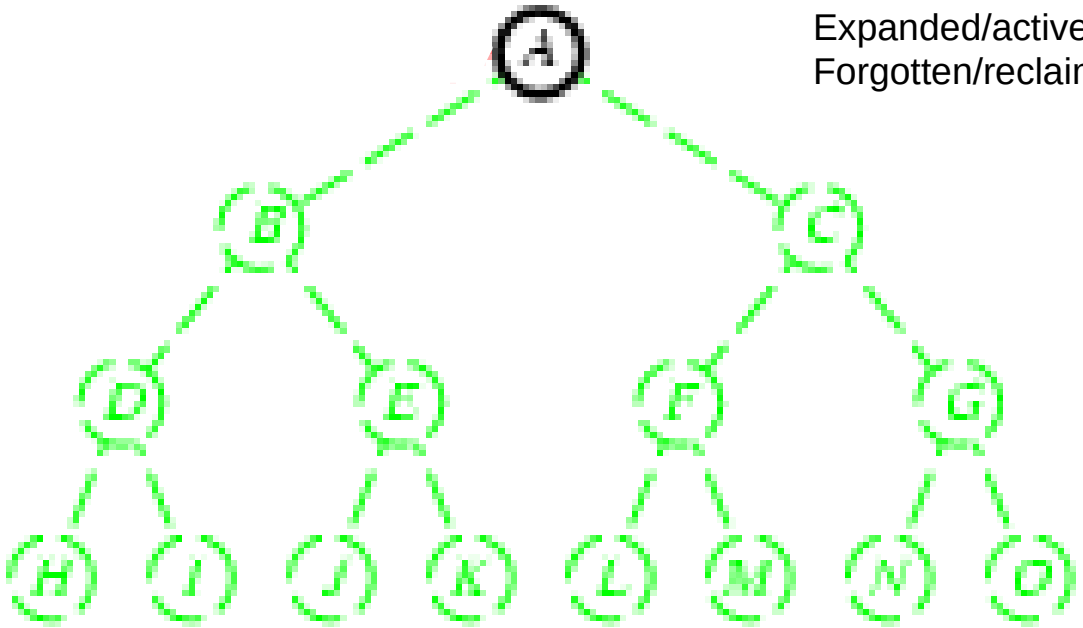
Backtracking search

- Similar to Depth-first search
  - At each level, picks a single variable to explore
  - Iterates over the domain values of that variable
- Generates kids one at a time, one per value
- Backtracks when a variable has no legal values left
- Uninformed algorithm
  - No good general performance

Backtracking search

- Expand *deepest* unexpanded node
- Generate ***only one*** child at a time.
- *Goal-Test* when inserted.
  - For CSP, Goal-test at bottom

Future= green dotted circles
Frontier=white nodes
Expanded/active=gray nodes
Forgotten/reclaimed= black nodes



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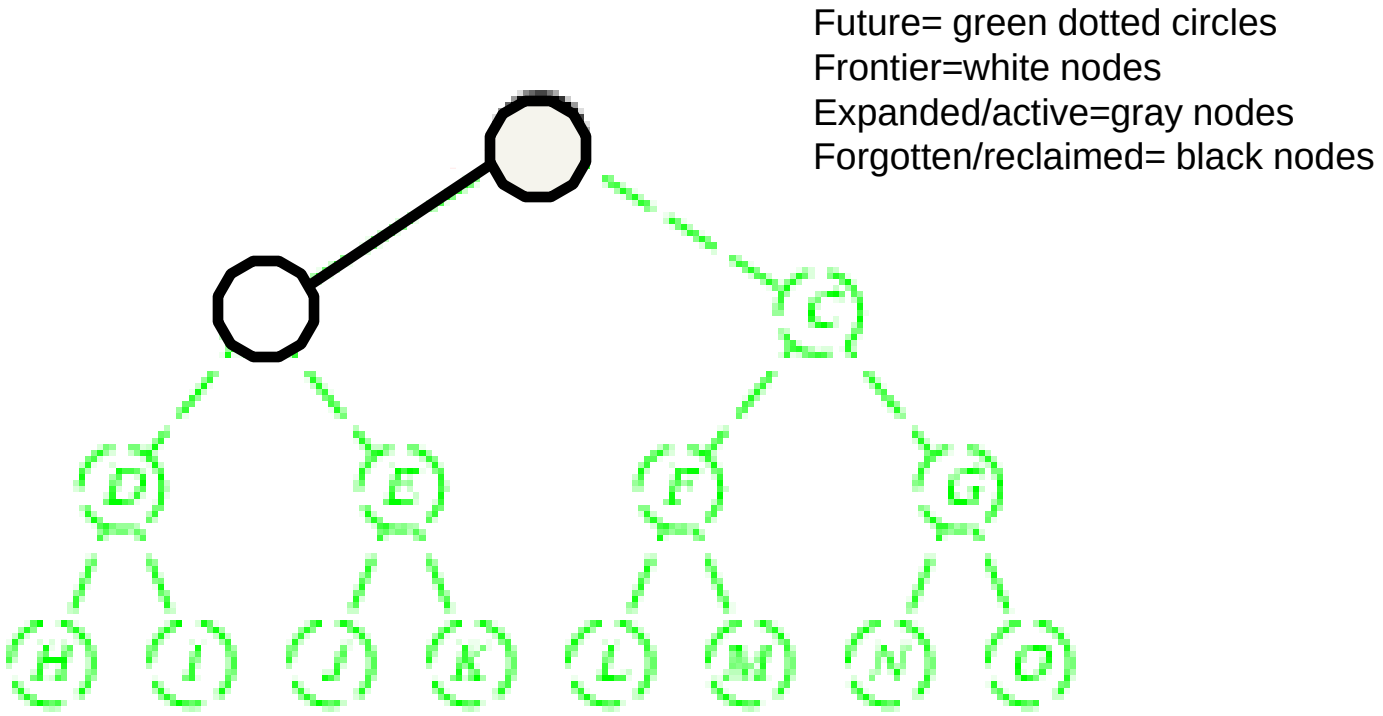

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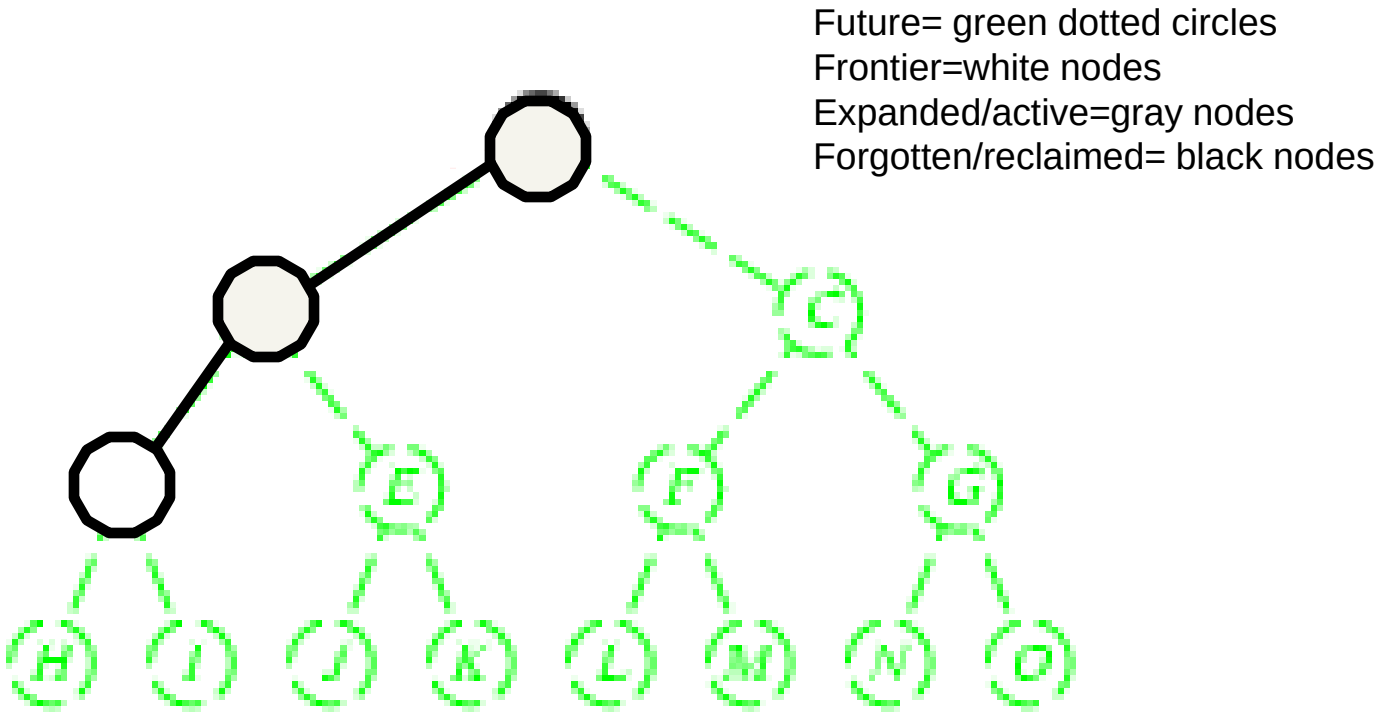

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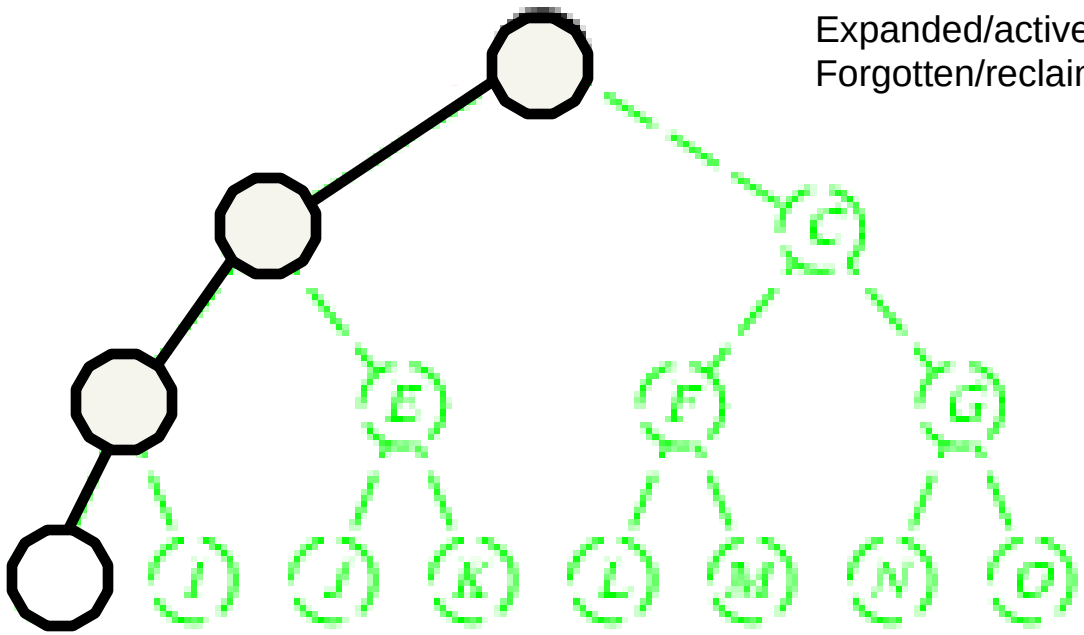

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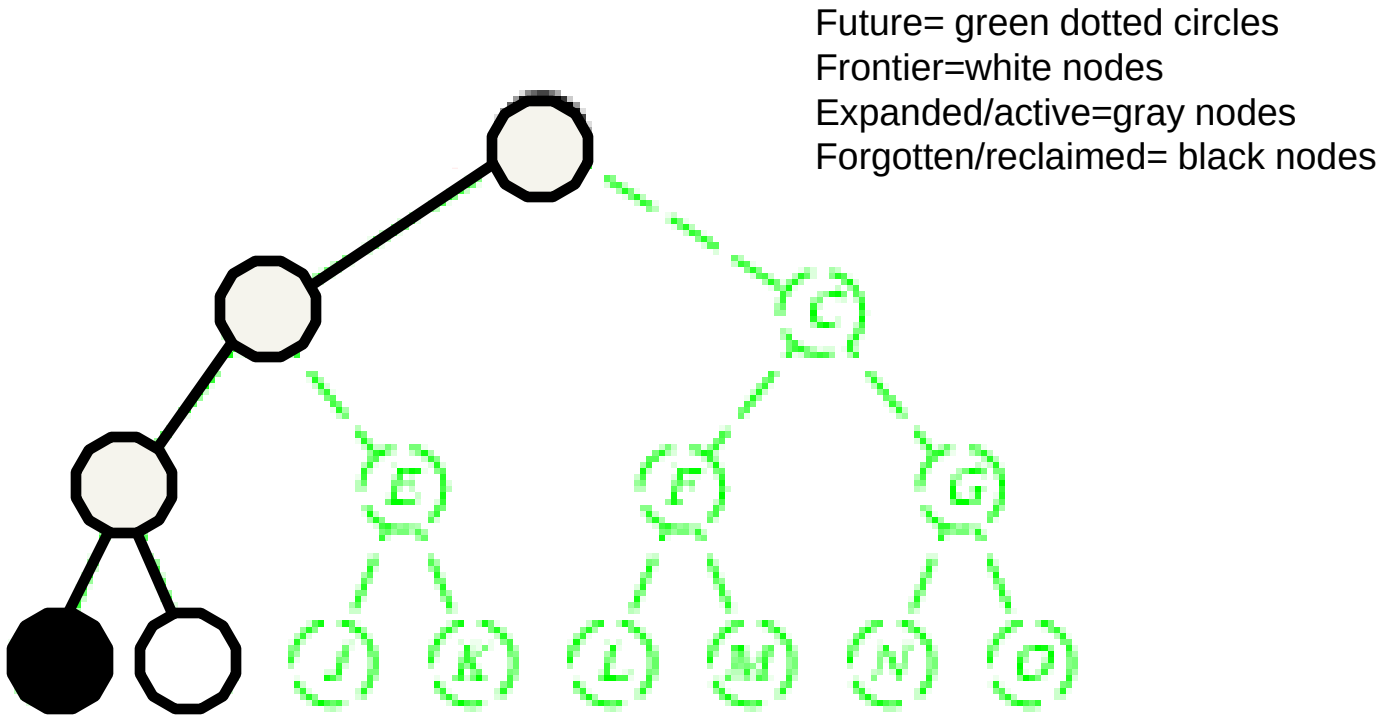

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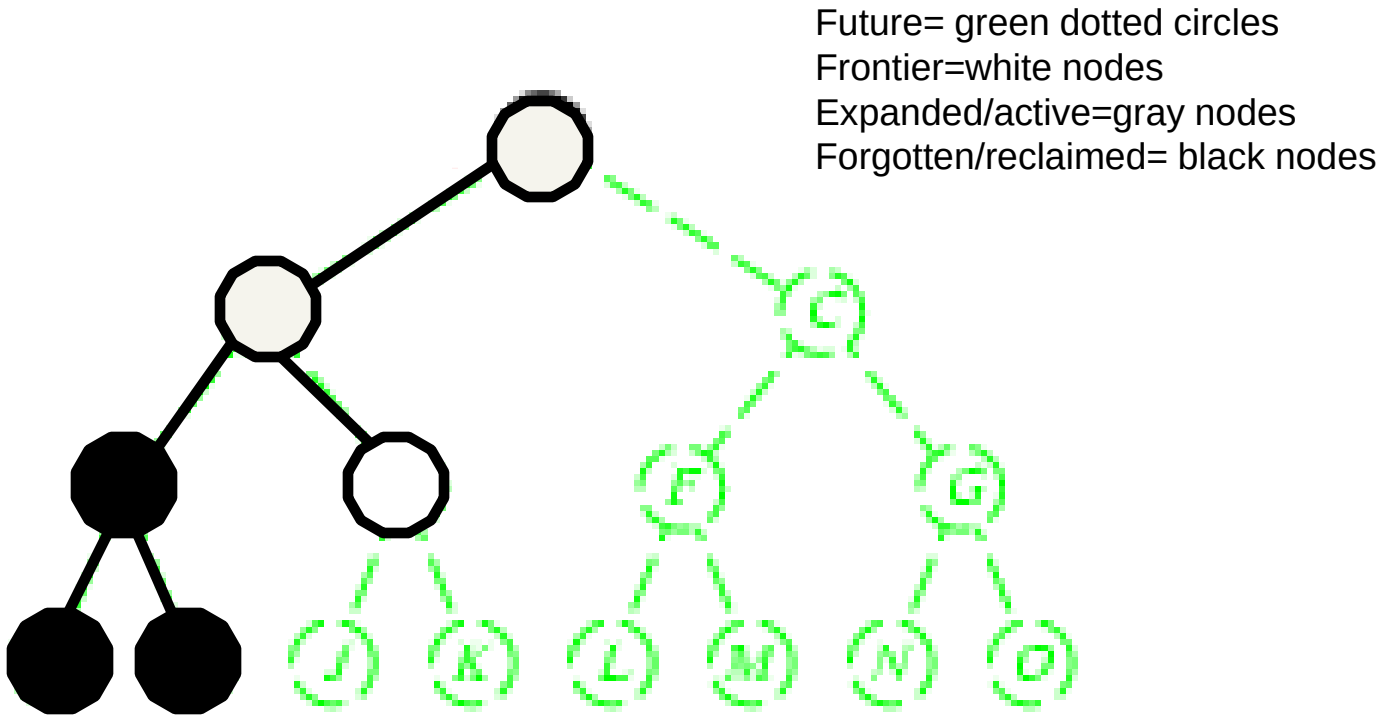

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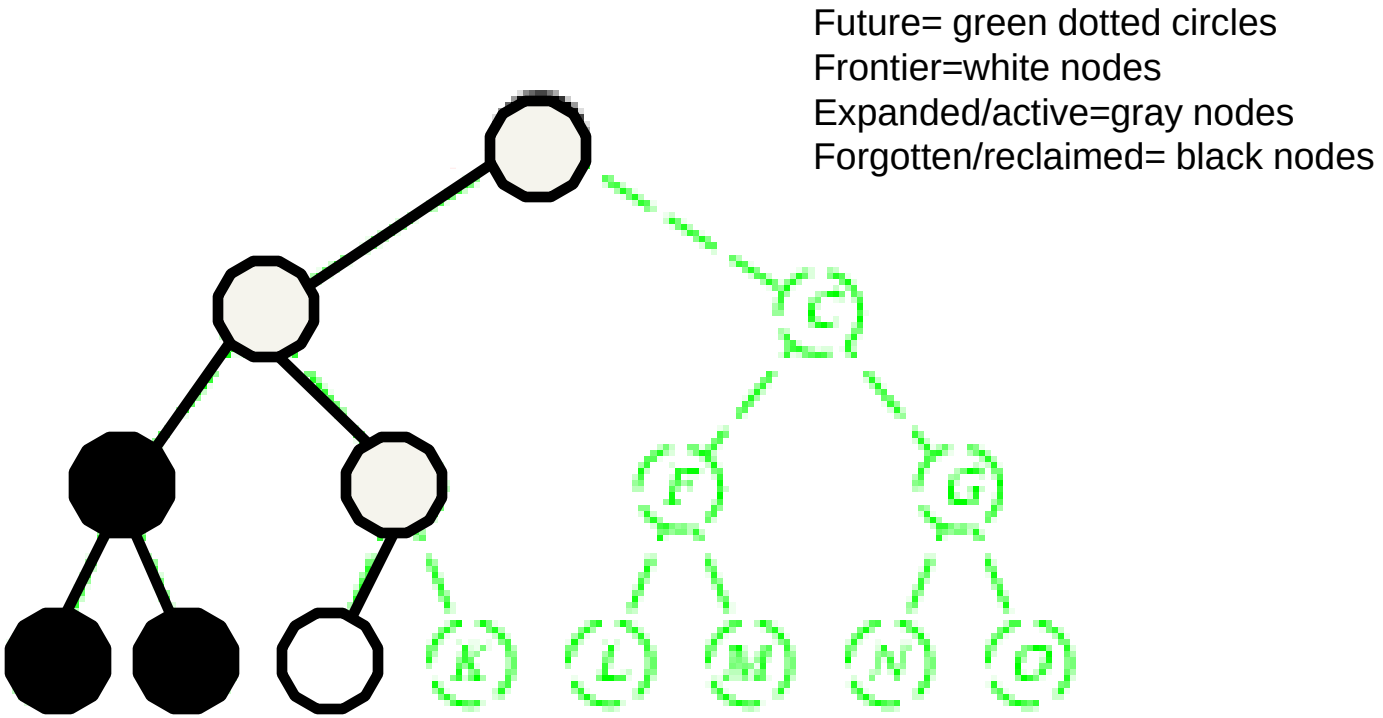

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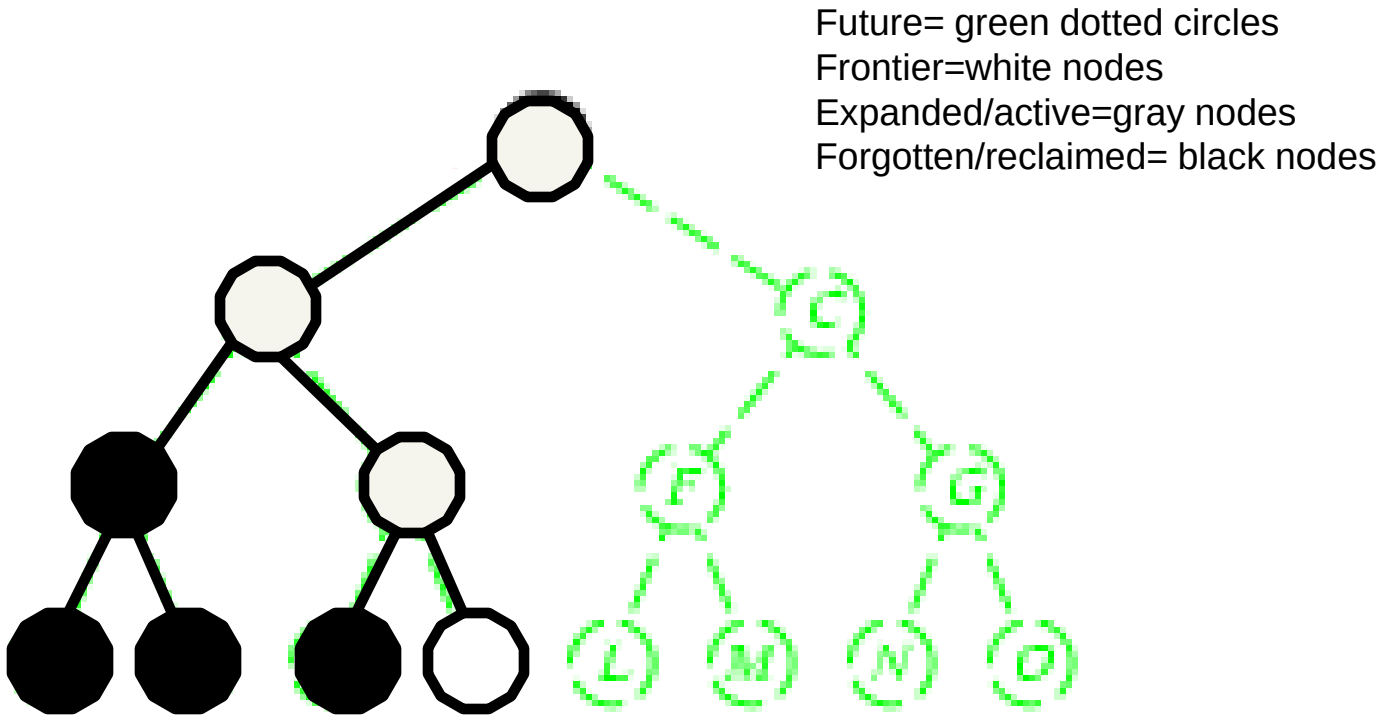

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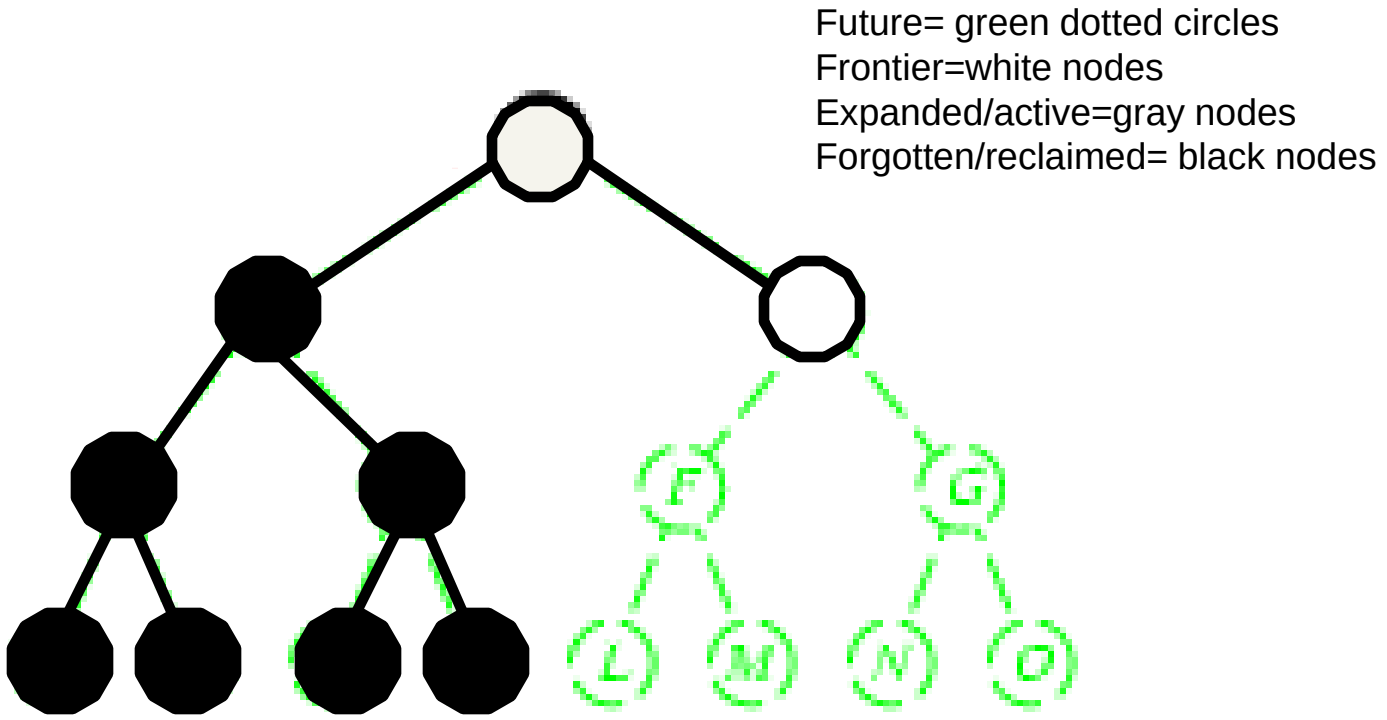

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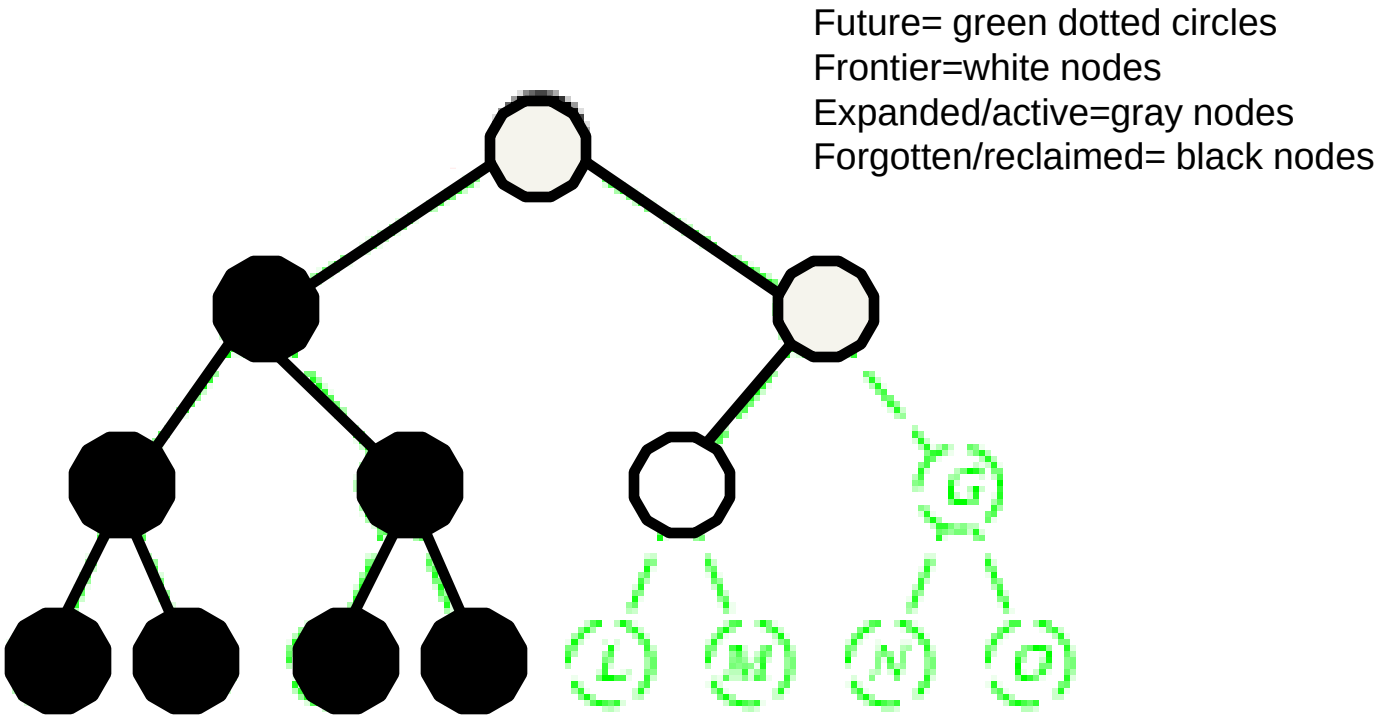

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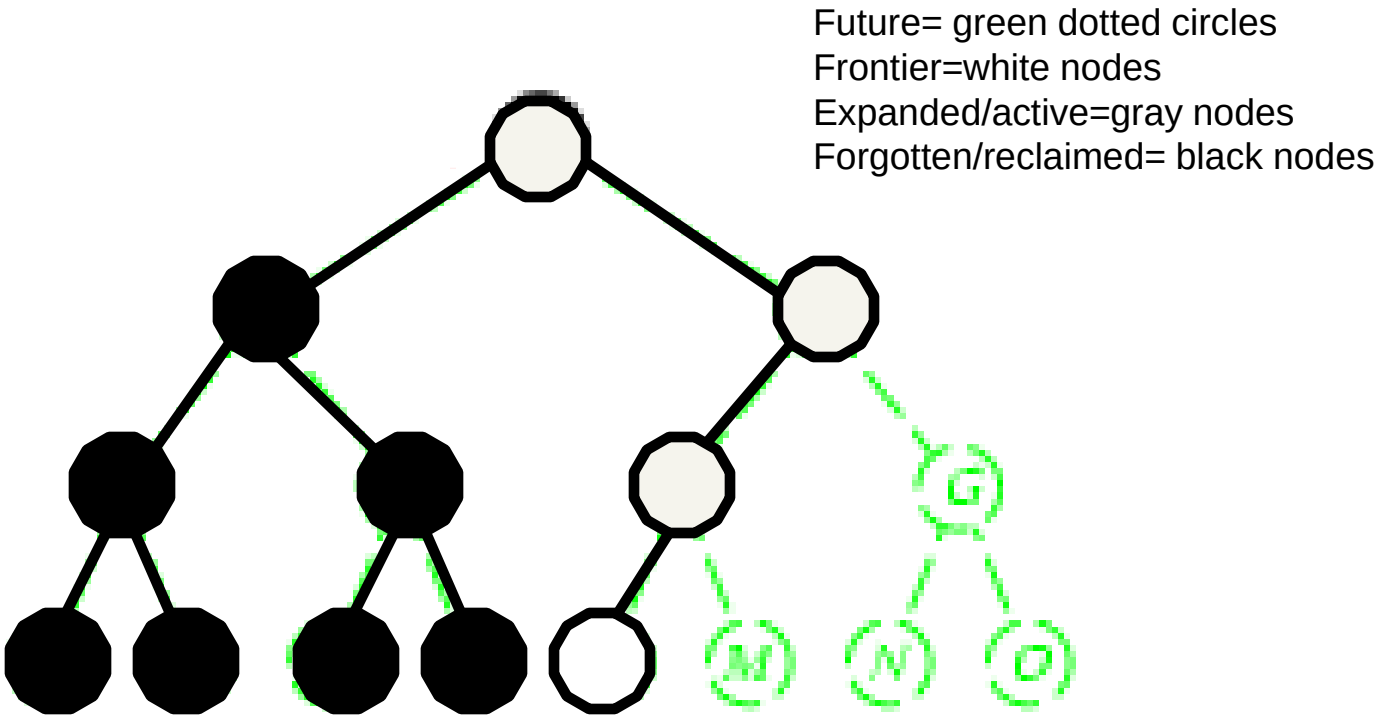

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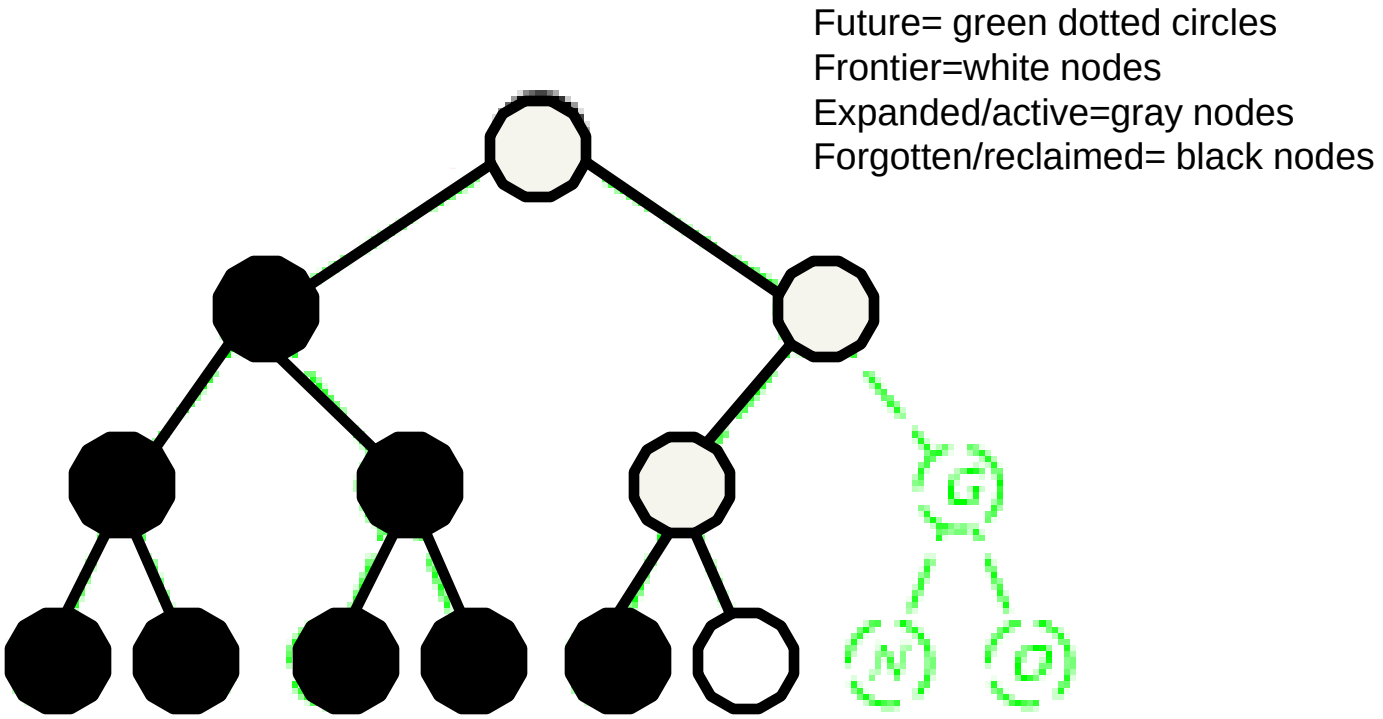


Backtracking search (Figure 6.5)

function BACKTRACKING-SEARCH(*csp*) **return** a solution or failure

    return RECURSIVE-BACKTRACKING(*{}* , *csp*)

function RECURSIVE-BACKTRACKING(*assignment, csp*) **return** a solution or failure

    if *assignment* is complete **then return** *assignment*

    var ← **SELECT-UNASSIGNED-VARIABLE(VARIABLES**[*csp*],*assignment*,*csp*)

    for each *value* **in ORDER-DOMAIN-VALUES**(*var, assignment, csp*) **do**

        if *value* is consistent with *assignment* according to CONSTRAINTS[*csp*] **then**

            add *{var=value}* to assignment

            result ← RECURSIVE-BACTRACKING(*assignment, csp*)

            if *result* ≠ *failure* **then return** *result*

            remove *{var=value}* from *assignment*

    return *failure*

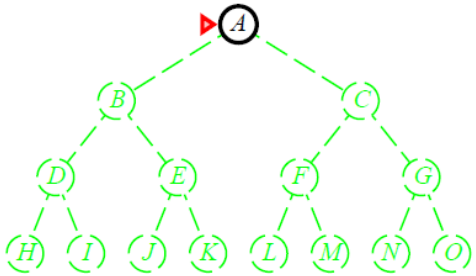
Depth First Search

Depth-first search

Expand deepest unexpanded node

Implementation:

fringe = LIFO queue, i.e., put successors at front



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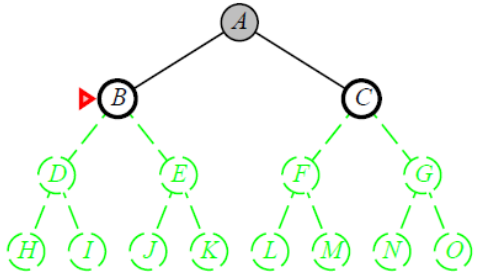

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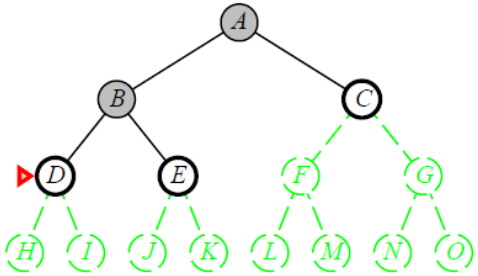

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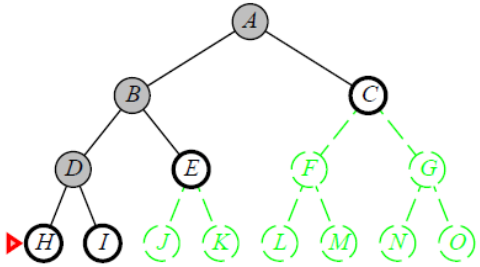

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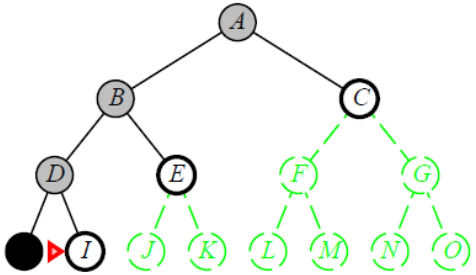

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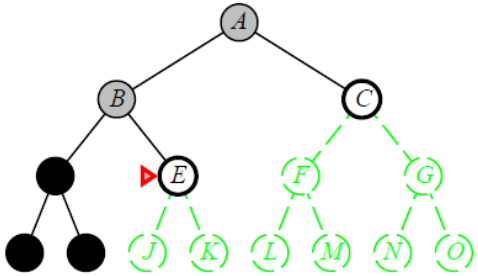

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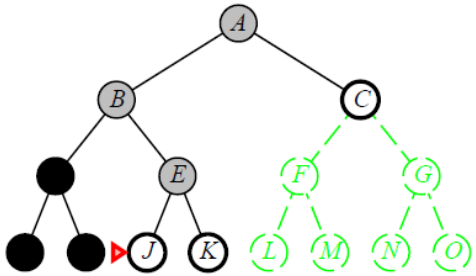

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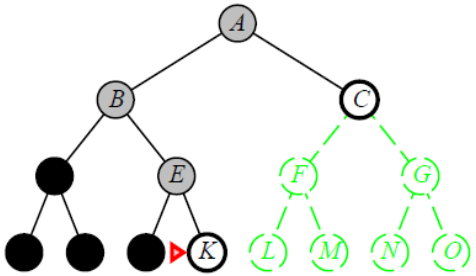

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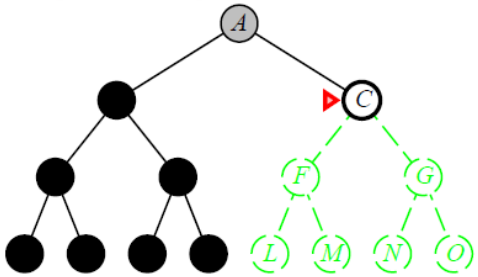

Improving CSP efficiency

- Previous improvements on uninformed search
  $\rightarrow$ introduce heuristics

- For CSPS, general-purpose methods can give large gains in speed, e.g.,
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?
  - Can we take advantage of problem structure?

Note: CSPs are somewhat generic in their formulation, and so the heuristics are more general compared to methods in Chapter 4

Heuristic

- Selecting Variable
  - Minimum remaining values (MRV)
    - choose variable with the fewest legal moves
  - Degree heuristic for next variable
    - select variable that is involved in the largest number of constraints on other unassigned variables
    - useful as a tie breaker after MRV.
- Selecting Value
  - Least constraining value (LCV)
    - given a variable choose the least constraining value

Backtracking search (Figure 6.5)

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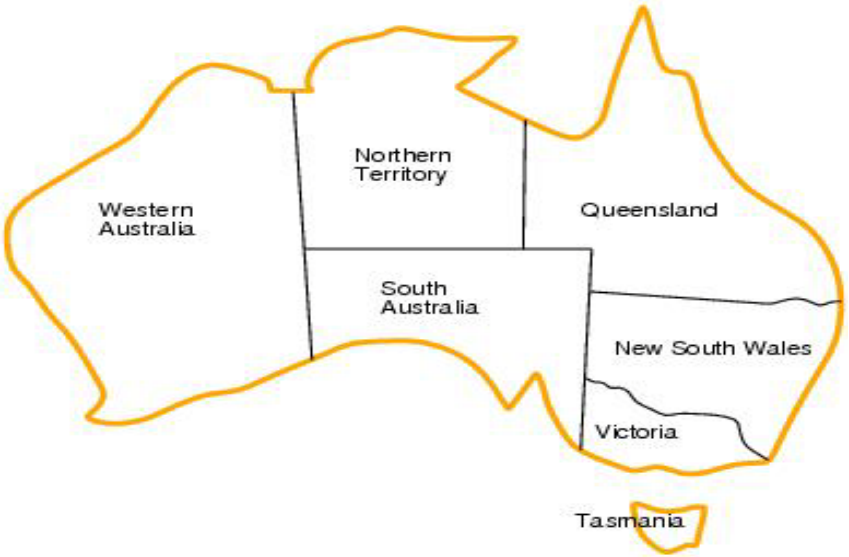
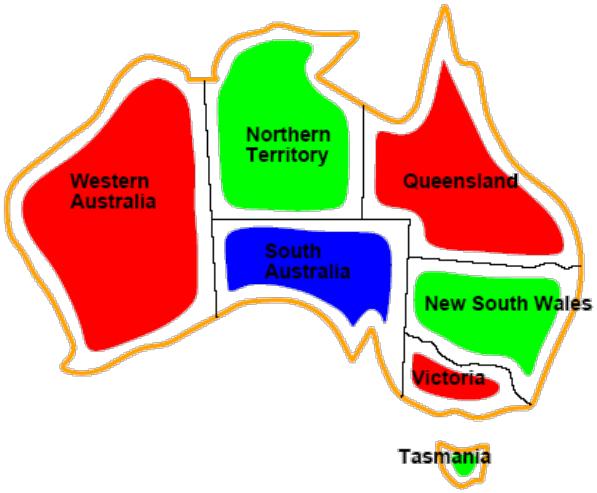
CSP example: Map coloring problem



- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: $D_i = \{red, green, blue\}$
- Constraints: adjacent regions must have different colors.
  - E.g. $WA \neq NT$

CSP example: Map coloring solution



- A solution is:
  - A complete and consistent assignment.
  - All variables assigned, all constraints satisfied.

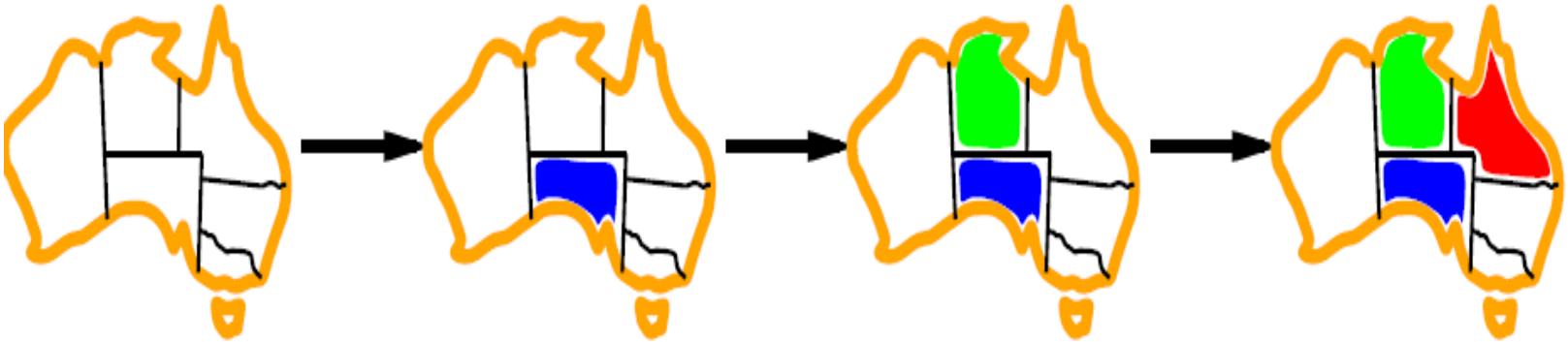
- E.g., *{WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}*

Minimum remaining values (MRV) for next variable

var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*],*assignment*,*csp*)

- A.k.a. most constrained variable heuristic
- *Heuristic Rule*: choose variable with the fewest legal moves
  - e.g., will immediately detect failure if X has no legal values

Degree heuristic for next variable



- *Heuristic Rule*: select variable that is involved in the largest number of constraints on other unassigned variables.
- Degree heuristic can be useful as a tie breaker after MRV.
- *In what order should a variable's values be tried?*

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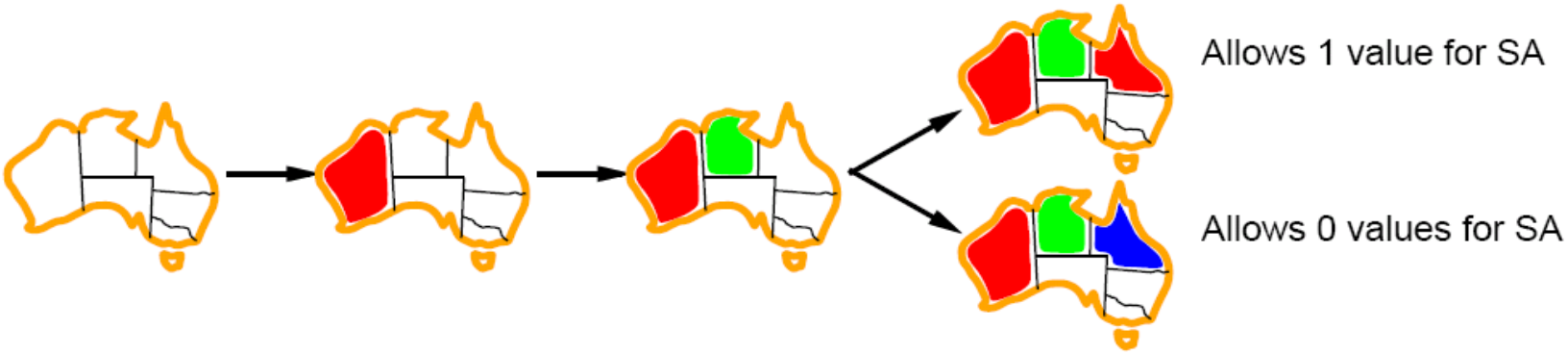
Least constraining value (LCV) for next value



- Least constraining value heuristic
- Heuristic Rule: given a variable choose the least constraining value
  - leaves the maximum flexibility for subsequent variable assignments

Minimum remaining values (MRV) vs. Least constraining value (LCV)

- Why do we want the MRV (minimum values, most constraining) for variable selection --- but the LCV (maximum values, least constraining) for value selection?

- Isn't there a contradiction here?

- MRV for variable selection to reduces the branching factor.
  - Smaller branching factors lead to faster search.
  - Hopefully, when we get to variables with currently many values, constraint propagation (next lecture) will have removed some of their values and they'll have small branching factors by then too.
- LCV for value selection increases the chance of early success.
  - If we are going to fail at this node, then we have to examine every value anyway, and their order makes no difference at all.
  - If we are going to succeed, then the earlier we succeed the sooner we can stop searching, so we want to succeed early.
  - LCV rules out the fewest possible solutions below this node, so we have the most chances for early success.