## ICS 171 - Quiz \#2 - TWENTY (20) minutes

1. ( 5 pts ) NAME AND EMAIL ADDRESS:

YOUR ID:___ ID TO RIGHT:___ NOW: FROM RIGHT: $\qquad$
2. ( 5 pts each, 30 pts total) Recall that

- True path cost so far $=g(n)$.
- Estimated cost to goal $=h^{\prime}(n)$.
- Estimated total cost $=f^{\prime}(n)=g(n)+h^{\prime}(n)$.

The following is a proof that best-first search (queue sorted by $f^{\prime}$ ) is optimal if the heuristic is admissable. The lines have been labelled A through G. Unfortunately, they have been scrambled.

Let $n_{g}$ be the first goal node popped off the queue. Let $n_{o}$ be any other node on the queue. We wish to prove that $n_{0}$ can never be extended to a path to any goal node that costs less than the path to $n_{g}$ that we just found.

$$
\begin{array}{ll}
A: \text { true total cost of } n_{g} \\
F: & =g\left(n_{g}\right) / / \text { because } n_{g} \text { represents a complete path } \\
D: & =f^{\prime}\left(n_{g}\right) / / \text { by definition of } f \text { with } h^{\prime}\left(n_{g}\right)=0 \\
B: & \leq f^{\prime}\left(n_{o}\right) / / \text { because queue is sorted by } f \\
E: & =g\left(n_{o}\right)+h^{\prime}\left(n_{o}\right) / / \text { by definition of } f \\
C: & \leq g\left(n_{o}\right)+\text { true cost to goal from } n_{o} / / \text { because } h^{\prime} \text { is admissable } \\
G: & =\text { true total cost of } n_{o}
\end{array}
$$

Fill in the blanks with the letters B, C, D, E, F , and G to prove that the true total cost of $n_{g} \leq$ true total cost of $n_{0}$. The first and last letters, A and G, have been done for you as an example.


Problem 3 asks about this graph. Assume that ALL children of a node are returned in alphabetical order whenever the node is expanded, and that ALL NODES ARE EXPANDED WHENEVER THEY APPEAR (this is different from the last quiz - keep track of everything). " S " is the start node, and either "G1" or "G2" are goal nodes. The number inside each node is an estimate of the remaining distance to any goal from that node. The number next to each arc is the operator cost for that arc.
3. (25 pts, -5 each wrong answer hut not negative)

Write the order in which best fi earch (sort queue by $f^{\prime}(n)=g(n)+h^{\prime}(n)$ ) expands nodes:

$$
\underline{\mathrm{S}} \xrightarrow{\mathrm{~A}} \xrightarrow{\mathrm{C}} \xrightarrow{\mathrm{~B}} \xrightarrow{\mathrm{~B}} \xrightarrow{\mathrm{G} 2}
$$

3. ( 5 pts each, 40 pts total) The "Eight Puzzle" is a classic AI search space that is often used to study search behavior. It reflects a simplified version of spatial planning and search under physical constraints, as in tasks such as laying out a factory floor. The puzzle consists of eight numbered tiles sealed into a $3 \times 3$ frame. Eight positions in the frame are occupied by tiles, and the ninth position is empty (the empty position is called the "blank").

The goal is to arrange the tiles in numerical order, as in the Goal State shown below. An operator in this search space is to choose a tile and push it into the blank, with the result that the tile and the blank switch positions. A state in this space is any legal arrangement of tiles, that is, any arrangement that can be reached by starting with the goal node and pushing the tiles around.
Goal State

| 1 | 2 | 3 |
| :---: | :---: | :---: |
| 8 |  | 4 |
| 7 | 6 | 5 |

Push " 4 " into blank

| 1 | 2 | 3 |
| :---: | :---: | :---: |
| 8 | 4 |  |
| 7 | 6 | 5 |

Typical Start State

| 1 |  | 3 |
| :---: | :---: | :---: |
| 7 | 2 | 5 |
| 4 | 8 | 6 |

3.a. Assume that each operator costs 1 (that is, that it costs 1 to push a tile into the blank). Let $\left(x_{i}, y_{i}\right)$ be the current position of tile $i$, and let $\left(x_{i}^{g}, y_{i}^{g}\right)$ be its goal position.

Which of the following heuristic functions are admissable, that is, NEVER yield a number that is greater than the minimum number of steps to the Goal State? ("Y" = Yes, "N" = No)
3.a.1. $\quad \mathbf{N} h^{\prime}=$ the number of CORRECTLY placed tiles (i.e., the number that ARE already in their goal positions).
3.a.2. $\quad \mathbf{Y} \quad h^{\prime}=$ the number of WRONGLY placed tiles (i.e., the number NOT already in their goal positions).
3.a.3. $\quad \mathbf{Y} h^{\prime}=$ the sum of the straight-line distances from each tile to its goal position. The straight-line distance from tile $i$ to its goal position is $\sqrt{\left(x_{i}-x_{i}^{g}\right)^{2}+\left(y_{i}-y_{i}^{g}\right)}$.
3.a.4. $\quad \mathbf{Y} h^{\prime}=$ the sum of the Manhattan distances (or city block distances) from each tile to its goal position. The Manhattan distance from tile $i$ to its goal position is $\left|x_{i}-x_{i}^{g}\right|+\left|y_{i}-y_{i}^{g}\right|$. 3.a.5. $\frac{\mathbf{Y}}{\mathbf{N}} \quad h^{\prime}=0$.
3.a.6. $\mathbf{N} \quad h^{\prime}=3$.
3.b. $\mathbf{4}$ (Answer with one of 1-6 above) Which one of the admissable heuristic functions in question 3.a above is best, i.e., which comes closest to the true remaining cost to the goal?
3.c. Recall that the branching factor is the average number of children per node. What is the branching factor $b$ for the eight puzzle? $\quad \mathbf{B}$
A. $b \leq 2$
B. $2<b \leq 4$
C. $4<b \leq 6$
D. $6 \leq b$

