

ICS 171 — Quiz #6 — TWENTY (20) minutes

1. (5 pts) NAME AND EMAIL ADDRESS: \_\_\_\_\_

YOUR ID: \_\_\_\_\_ ID TO RIGHT: \_\_\_\_\_ ROW: \_\_\_\_\_ NO. FROM RIGHT: \_\_\_\_\_

2. (5 pts) Write down the definition of  $P(H|D)$  in terms of  $P(H)$ ,  $P(D)$ ,  $P(H \text{ and } D)$ , and  $P(H \text{ or } D)$ .

$$P(H|D) = \frac{P(H \text{ and } D)}{P(D)}$$

3. (5 pts) Write down the expression that results from applying Bayes' Rule to  $P(H|D)$ .

$$P(H|D) = P(D|H) \frac{P(H)}{P(D)}$$

4. (5 pts) Write down the definition of  $A \rightarrow B$  in terms of “and”, “or”, and “not”.

$$A \rightarrow B = ((\text{not } A) \text{ or } B)$$

(Other equivalent logical operators are OK.)

5. (5 pts each, 30 pts total) Mark the following statements as T (= true) or F (= false).

a.     **T**      $P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$

b.     **T**      $P(A \text{ and } B) = P(A|B)P(B)$

c.     **T**      $P(A \text{ and } B) = P(A)P(B)$  if and only if A and B are independent.

d.     **F**      $P(A \text{ or } B) = P(A) + P(B)$  if and only if A and B are independent.

e.     **F**      $P(A \text{ and } B) = P(A)P(B)$  if and only if A and B are disjoint (do not intersect, or do not occur together).

f.     **T**      $P(A \text{ or } B) = P(A) + P(B)$  if and only if A and B are disjoint (do not intersect, or do not occur together).

6. (5 pts each, 40 pts total) Let  $PKF(x, y)$  mean “Person  $x$  Knows Fact  $y$ ”. For purposes of this question only, you may assume that the first argument is a person and the second is a fact.

For each English sentence below, write the logic sentence that best expresses it. Use “ $\neg$ ” to mean “not.” The first one is done for you.

- a. Every person knows every fact.  $\forall x \forall y \ PKF(x, y).$
- b. Every person knows at least one fact.  $\forall x \exists y \ PKF(x, y).$
- c. There is a person who knows at least one fact.  $\exists x \exists y \ PKF(x, y).$
- d. There is a person who knows every fact.  $\exists x \forall y \ PKF(x, y).$
- e. No person knows every fact.  
Equivalent:  $\neg \exists x \forall y \ PKF(x, y).$   
 $\forall x \exists y \ \neg PKF(x, y).$
- f. There is a person who knows no fact.  $\exists x \forall y \ \neg PKF(x, y).$
- g. No person knows any fact.  $\forall x \forall y \ \neg PKF(x, y).$
- h. There is a fact that is known by every person.  $\exists y \forall x \ PKF(x, y).$
- i. There is a fact that no person knows.  
Equivalent:  $\exists y \neg \exists x \ PKF(x, y).$   
 $\exists y \forall x \ \neg PKF(x, y).$

7. (2 pts each, 10 pts total) Fill in each blank below with Y (= Yes) or N (= No) depending on whether the logic expression correctly expresses the English.

- a.     N     “All cats are mammals.”  $\forall x \text{Cat}(x) \& \text{Mammal}(x)$   
 “All cats are mammals.” is  $\forall x \text{Cat}(x) \Rightarrow \text{Mammal}(x).$   
 $\forall x \text{Cat}(x) \& \text{Mammal}(x)$  is “Everything is a cat and a mammal.”
- b.     Y     “Spot has a sister who is a cat.”  $\exists x \text{Sister}(x, \text{Spot}) \& \text{Cat}(x)$
- c.     N     “For every person, there is someone that that person likes.”  $\exists x \forall y \text{Likes}(x, y)$   
 “For every person, there is someone that that person likes.” is  $\forall x \exists y \text{Likes}(x, y).$   
 $\exists x \forall y \text{Likes}(x, y)$  is “There is someone who likes everyone.”
- d.     N     “There is someone who is liked by everyone.”  $\forall x \exists y \text{Likes}(x, y)$   
 “There is someone who is liked by everyone.” is  $\exists y \forall x \text{Likes}(x, y)$   
 $\forall x \exists y \text{Likes}(x, y)$  is “For every person, there is someone that that person likes.”
- e.     Y     “Everyone likes ice cream.”  $\neg \exists x \neg \text{Likes}(x, \text{IceCream})$