Data Acquisition for Embedded Systems

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Overview

- Signal Processing Basics
- Sampling
- Analog-to-Digital Conversion
- Digital-to-Analog Conversion
Signals and Systems

- **Signal**: a set of information and data
  - e.g. telephone, television signal, daily stock quote
  - Continuous and Discrete-time
  - Analog and Digital

- **System**: an entity that processes a set of signals (inputs) to yield another set of signals (outputs)
  - **Characteristics**: inputs, outputs, rules of operation
  - **Classification**:
    - Linear and non-linear
    - Constant parameter and time-varying parameter
(Trigonometric) Fourier Series

For a periodic function, \( f(t) \), with period, \( T = 1 / f_0 \),

\[
f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(2 \pi f_0 kt) + \sum_{k=1}^{\infty} b_k \sin(2 \pi f_0 kt)
\]

\[
= C_0 + \sum_{k=1}^{\infty} C_k \cos(2 \pi f_0 kt + \theta_k)
\]

\( f_0 \) = Fundamental frequency, \( n \cdot f_0 = n^{th} \) harmonic of \( f_0 \)

\( a_k, b_k \) = Amplitude of various harmonics

\[
a_0 = C_0 = \frac{1}{T} \int_T f(t) \, dt
\]

\[
a_k = \frac{2}{T} \int_T f(t) \cos(2 \pi f_0 kt) \, dt, \quad b_k = \frac{2}{T} \int_T f(t) \sin(2 \pi f_0 kt) \, dt
\]

\[
C_n = \sqrt{a_k^2 + b_k^2}
\]
Fourier Transform

- Even if a function is not periodic, it can be described as a linear combination of an infinite number of orthogonal functions (In case of Fourier Transform, sinusoids). i.e. spectrum consists of a continuum of frequencies.

\[ (T \to \infty, f_0 \to 0) \]

- This spectrum can be defined by Fourier transform.

- For a signal \(x(t)\) with a spectrum \(X(f)\), the followings hold:

\[
X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt \quad \text{(Fourier Transform)}
\]

- \(\langle x(t), X(f) \rangle\): Fourier transform pairs

\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, df \quad \text{(Inverse Fourier Transform)}
\]
Time vs. Frequency Domain Representation

\[ f(t) = \frac{8A}{\pi^2} \cos(\pi t - 90^\circ) + \frac{1}{9} \cos(3\pi t + 90^\circ) + \frac{1}{25} \cos(5\pi t) \]

\[ x(t) \]

\[ X(f) \]

ICS180

Prof. Veidenbaum
Sampling

- Sampling is a method of converting a continuous signal to a discrete set of values (samples)
  - should accurately represent the original signal
  - reduce the amount of information to be processed
- Major Concern:
  - How fast a given continuous signal must be sample to accurately represent the original
- Aliasing:
  - Frequency ambiguity \([f_0 \text{ vs. } (f_0 + k f_s), \ k=\text{integer}]\)
Sampling

Examples below:

1) samples a slow (1KHz) signal with a 7KHz sine signal
2) samples a -2KHz signal with a 4KHz sine signal
Typical Sampling Rate

Sample Rate (Hz)

Less Complex

Algorithm Complexity

More Complex

10G
1G
100M
10M
1M
100k
10k
1k
100
10
1
1/10
1/100
1/1000

Radio Signaling and Radar
High Definition Television
Video
Audio
Speech
Control
Instrumentation
Radio Modems
Voiceband Modems
Seismic Modeling
Financial Modeling
Weather Modeling
Sampled Signal & Fourier Spectrum

\[
\delta_T(t) = \frac{1}{T} \left[ 1 + 2 \cos 2\pi F_s t + 2 \cos 4\pi F_s t + 2 \cos 6\pi F_s t + \cdots \right]
\]

\[
\overline{F}(w) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_s)
\]
Nyquist Rate

- A signal which is (spectrum) band-limited to $B$ Hz can be reconstructed exactly from its samples taken uniformly at a rate $R > 2B$ samples per second.
  - This is called Nyquist rate for the signal.

- Practical Difficulty
  - Low-pass filter limitation
  - Spectra should consist of repetitions of $F(w)$ with a finite gap between successive cycles
Aliasing Effect

- All practical signals are time-limited
  - (non-band-limited)
- The spectrum $\tilde{F}(\omega)$ consists of overlapping cycles of repeating every $F_s$ Hz
  - No longer complete recovery
- Aliasing (Spectral folding)
  - Folding frequency $= 1/2$ sampling frequency
Elimination of Aliasing: Anti-Aliasing Filter

- Band-limited signal: No alias if $F_s > \text{Nyq. Rate}$
- Non-band-limited signal:
  - Alias results regardless of sampling rate
- Aliasing can be eliminated by band-limiting a signal before sampling
- Anti-aliasing filter: low-pass filter of bandwidth $B$ Hz ($B$: effective bandwidth)
### Analog vs. Digital - Pro’s

<table>
<thead>
<tr>
<th>Analog</th>
<th>Digital</th>
</tr>
</thead>
<tbody>
<tr>
<td>- High bandwidth</td>
<td>- Programmable solution</td>
</tr>
<tr>
<td>- High resolution</td>
<td>- Less sensitive to environment</td>
</tr>
<tr>
<td>- Specific control functions are available as off-the-shelf ICs</td>
<td>- Can implement advanced control algorithms</td>
</tr>
<tr>
<td>- Analysis and design methods are well-known</td>
<td>- Capable of self-tuning, adaptive control, and nonlinear control functions</td>
</tr>
<tr>
<td></td>
<td>- Communication capability</td>
</tr>
</tbody>
</table>
## Analog vs. Digital - Con’s

<table>
<thead>
<tr>
<th>Analog</th>
<th>Digital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature drift</td>
<td>Data converter is required.</td>
</tr>
<tr>
<td>Component aging</td>
<td>Analysis and design methods are more complex</td>
</tr>
<tr>
<td>Sensitive to noise</td>
<td>Sampling &amp; quantization error</td>
</tr>
<tr>
<td>Hardware design</td>
<td>Computation delay limits the system bandwidth</td>
</tr>
<tr>
<td>Can implement simple designs only</td>
<td></td>
</tr>
<tr>
<td>No communication capability</td>
<td></td>
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General Considerations

- Resolution: the number of bits in the input
- Figures of merit
  - resolution
  - temperature sensitivity
  - linearity
  - absolute accuracy (quantization error)
  - settling time, conversion time
  - price, complexity
- Errors
  - Static errors: Offset errors, Gain errors, Nonlinearity, Nonmonotonicity
  - Dynamic errors: degradation of S/N, glitches
Errors

static error

glitch
Analog to Digital and Digital to Analog Conversion

- Used in Embedded Systems to interact with physical world
- A-to-D is the nexus of data acquisition process
  - ADC has been typically, the slowest, most complex, and/or most expensive single component in the data path

Primary Concerns

- Speed of Operation
- Resolution (# of bits at the output)
- Linearity, Non-monotonicity
- Cost, Size, Power requirements, etc.
### How fast should ADC be?

<table>
<thead>
<tr>
<th>Applications</th>
<th>Approx. No of conversions per second</th>
<th>required conversion time</th>
</tr>
</thead>
<tbody>
<tr>
<td>monitor and control</td>
<td>1 - 1000</td>
<td>15 - 1 ms</td>
</tr>
<tr>
<td>telephone voice</td>
<td>8,000</td>
<td>125 ms</td>
</tr>
<tr>
<td>CD-quality audio</td>
<td>85, 42.5, 21.3 K</td>
<td>50 - 12 ms</td>
</tr>
<tr>
<td>video</td>
<td>1 - $10^6$</td>
<td>100 ns - 1 ms</td>
</tr>
<tr>
<td></td>
<td>3 x 1 - $10^6$</td>
<td></td>
</tr>
<tr>
<td>radar</td>
<td>100 - 1,000 x $10^6$</td>
<td>1 - 10 ns</td>
</tr>
</tbody>
</table>
Common DAC Structures

- Weighted Resistor DAC
  - N resistors and switches (transistors)

- R/2R ladder DAC
Simple DAC

- Weighted resistor implementation
- Very stable $V_{ref}$ required
- Straightforward in concept
- Impractical because a large resistor range is required
- Practically, can be up to 8 bits

If the content of the register represents the number $\sum_{k=0}^{n-1} a_k 2^{-k}$,

- $R_0 = R$, $R_1 = 2R$, $R_2 = 4R$, ..., $R_{n-1} = 2^{n-1}R$
- $I_0 = I$, $I_1 = \frac{I}{2}$, $I_2 = \frac{I}{4}$, ..., $I_{n-1} = \frac{I}{2^{n-1}}$
R/2R Ladder

- A major improvement on the previous design
  - Use only two distinct resistor values
  - Output resistance is constant, independent of stages, $n$
  - Fully modular - stages can be added or deleted w/o compromising the design
- Each digit $I$ adds $\sim V/I$ volts to the output sum
- Rather slow
R/2R Ladder DAC

Digital inputs (controlling the switches)

Vref

R

R

... 

2R

2R

2R

2R

2R

R

0

0

Vout (Buffered)

Current-valued output

Voltage-valued output

n
Digital to Analog Conversion

- Convert each bit (digit) to a weighted voltage level, sum them
- Can use A-to-D to perform D-to-A.
Common ADC Structures

- Successive-Approximation (serial) ADC - medium
  - 12-16 bits
  - 1-10 micro seconds - slow
  - Various forms of approximation:
    - simple counter
    - initial guess and up/down count
    - digit by digit, starting at msb

- Parallel ("Flash") ADC - fast, but very expensive
  - uses a resistive "divider" to generate reference voltage for each bit
  - <= 8 bits
  - fast
Successive Approximation

- Conversion algorithm
  - Start with an initial guess
  - Output the guess to DAC, compare the result of DAC to $V_{in}$
  - Improve the guess

- Analogy:

![Diagram of a balance scale with voltages and binary values]
A-to-D via Counting or Successive Approximation
Flash(Parallel) ADC- fastest

Conversion speed: 4-50ns
Resolution: limited to 8 bits due to chip area and dc power dissipation
Complexity: exponential to n

Comparator

analog input

Vref

Priority encoder

n-bit Digital output

sample
Summary

- Sampling of analog signals and digital conversion
  - Sampling Theorem
    - Nyquist rate
    - Aliasing
  - Digital to Analog Conversion
    - R/2R
  - Analog to Digital Conversion
    - Flash
    - Successive Approximation