## Announcements

Homework 1 is released

- Available on the course website
- Due in two weeks: 10/22/19 11:59pm

Submit through GradeScope

- TA Sam gave a tutorial last Wednesday


## Lecture 4

## Encryption II

## Suggested Readings:

- Chs 3 \& 4 in KPS (recommended)
- Ch 3 in Stinson (optional)


## Conventional (Symmetric) Cryptography



## "Modern" Block Ciphers

## Data Encryption Standard (DES)



## Function F



## DES Substitution Boxes Operation



# Operation Tables of DES (IP, IP-1,$E$ and $P$ ) 

Initial Pemutation (IP)

| 58 | 50 | 42 | 34 | 26 | 18 | 10 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | 52 | 44 | 36 | 28 | 20 | 12 | 4 |
| 62 | 54 | 46 | 38 | 30 | 22 | 14 | 6 |
| 64 | 56 | 48 | 40 | 32 | 24 | 16 | 8 |
| 57 | 49 | 41 | 33 | 25 | 17 | 9 | 1 |
| 59 | 51 | 43 | 35 | 27 | 19 | 11 | 3 |
| 61 | 53 | 45 | 37 | 29 | 21 | 13 | 5 |
| 63 | 55 | 47 | 39 | 31 | 23 | 15 | 7 |

Inverse Initial Pemutation ( $\mathrm{IP}^{-1}$ )

| 40 | 8 | 48 | 16 | 56 | 24 | 64 | 32 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 39 | 7 | 47 | 15 | 55 | 23 | 63 | 31 |
| 38 | 6 | 46 | 14 | 54 | 22 | 62 | 30 |
| 37 | 5 | 45 | 13 | 53 | 21 | 61 | 29 |
| 36 | 4 | 44 | 12 | 52 | 20 | 60 | 28 |
| 35 | 3 | 43 | 11 | 51 | 19 | 59 | 27 |
| 34 | 2 | 42 | 10 | 50 | 18 | 58 | 26 |
| 33 | 1 | 41 | 9 | 49 | 17 | 57 | 25 |

Bit-Selection Table E

| 32 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5 | 6 | 7 | 8 | 9 |
| 8 | 9 | 10 | 11 | 12 | 13 |
| 12 | 13 | 14 | 15 | 16 | 17 |
| 16 | 17 | 18 | 19 | 20 | 21 |
| 20 | 21 | 22 | 23 | 24 | 25 |
| 24 | 25 | 26 | 27 | 28 | 29 |
| 28 | 29 | 30 | 31 | 32 | 1 |

Permutation $P$

| 16 | 7 | 20 | 21 |
| :---: | :---: | :---: | :---: |
| 19 | 12 | 18 | 17 |
| 1 | 15 | 23 | 26 |
| 5 | 18 | 31 | 10 |
| 2 | 8 | 24 | 14 |
| 32 | 27 | 3 | 9 |
| 19 | 13 | 30 | 6 |
| 22 | 11 | 4 | 25 |



Key schedule of shifts

| Iteration(i)i | No. of shifts |
| :---: | :---: |
| 1 | 1 |
| 2 | 1 |
| 3 | 2 |
| 4 | 2 |
| 5 | 2 |
| 6 | 2 |
| 7 | 2 |
| 8 | 2 |
| 9 | 1 |
| 10 | 2 |
| 11 | 2 |
| 12 | 2 |
| 13 | 2 |
| 14 | 2 |
| 15 | 2 |
| 16 | 1 |

Key permutation PC-1

| 57 | 49 | 41 | 33 | 25 | 17 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 58 | 50 | 42 | 34 | 26 | 18 |
| 10 | 2 | 59 | 51 | 43 | 35 | 27 |
| 19 | 11 | 3 | 60 | 52 | 44 | 36 |
| 63 | 55 | 47 | 39 | 31 | 23 | 15 |
| 7 | 62 | 54 | 46 | 38 | 30 | 22 |
| 14 | 6 | 61 | 53 | 45 | 37 | 29 |
| 21 | 13 | 5 | 28 | 20 | 12 | 4 |

*) Key permutation PC-2

| 14 | 17 | 11 | 24 | 1 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 28 | 15 | 6 | 20 | 10 |
| 23 | 19 | 12 | 4 | 26 | 8 |
| 16 | 7 | 27 | 20 | 13 | 2 |
| 41 | 52 | 31 | 37 | 47 | 55 |
| 30 | 40 | 51 | 45 | 33 | 48 |
| 44 | 49 | 39 | 56 | 34 | 54 |
| 46 | 42 | 50 | 36 | 29 | 32 |

Operation Tables of DES (Key Schedule,PC-1,PC-2)

## Breaking DES (Cryptanalysis)

DES Key size = 56 bits

- Brute force $=2^{55}$ attempts on avg
- Differential cryptanalysis $\rightarrow 2^{47}$ chosen plaintexts [BS'89]
- Linear cryptanalysis $\rightarrow 2^{43}$ known plaintexts [M'93]
- More than 16 rounds do not make it any stronger
-DES Key Problems:
-Weak keys (all 0s, all 1s, a few others)
- Key size $=56$ bits $=8$ * 7-bit ASCII
-Alphanumeric-only password converted to uppercase

$$
8 * \sim 5 \text {-bit chars }=40 \text { bits }
$$

# Modes of Operation <br> (not just for DES, for any block cipher) 

| $P_{1}$ | $P_{2}$ | $\ldots$ | $P_{1}$ | $P_{n+1}$ | $\ldots$ | $P_{n-1}$ | $P_{n}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## ENCRYPTION

$\mathrm{C}_{1} \mathrm{C}_{2}$
C
C
$i+1$
-••

http://en.wikipedia.org/wiki/Block_cipher_mode_of_operation

## "Native" ECB Mode

## Electronic Code-Book (ECB) Mode

-Input to encryption algorithm is current plaintext block:

$$
\begin{aligned}
& C_{i}=E\left(K, P_{i}\right) \\
& P_{i}=D\left(K, C_{i}\right)
\end{aligned}
$$

-Duplicate plaintext blocks (patterns) visible in ciphertext

- What if Alice encrypts one word per plaintext block?
-Ciphertext block rearrangement is possible
- To detect it, need explicit block numbering in plaintext
-Parallel encryption and decryption (random access)
-Error in one ciphertext block $\rightarrow$ one-block loss
$\bullet$-One-block loss in ciphertext?


## CBC Mode

## Cipher-Block Chaining (CBC) Mode

-Input to encryption algorithm is the XOR of current plaintext block and preceding ciphertext block:

$$
\begin{gathered}
C_{i}=E\left(K, P_{i} \text { XOR } C_{i-1}\right) \quad C_{0}=I V \\
P_{i}=D\left(K, C_{i}\right) \text { XOR } C_{i-1}
\end{gathered}
$$

-Duplicate plaintext blocks (patterns) NOT exposed
-Block rearrangement is detectable

- No parallel encryption
- How about parallel decryption?
$\bullet$ Error in one ciphertext block $\rightarrow$ two-block loss
-One-block ciphertext loss?


Figure 2.7 Cipher Block Chaining (CBC) Mode

## OFB Mode

## Output Feedback (OFB) Mode

- Key-stream is produced by repeated encryption of $\mathrm{V}_{0}$ :

$$
\begin{gathered}
C_{i}=E\left(K, V_{i-1}\right) \text { XOR }_{i} \quad V_{0}=I V, \ldots, V_{i}=E\left(K, V_{i-1}\right) \\
P_{i}=E\left(K, V_{i-1}\right) X O R C_{i}
\end{gathered}
$$

- Duplicate plaintext blocks (patterns) NOT exposed
- Block rearrangement is detectable
- Key-stream is independent of plaintext
- How does that affect speed of encryption? Parallelism?
- Bit error in one ciphertext block $\rightarrow$ one-bit error in plaintext
- One-block ciphertext loss $\rightarrow$ big mess $)$
- Can encrypt less than block size


## CFB Mode

## Cipher Feedback (CFB) Mode

$\cdot$ Key-stream is produced by re-encryption of preceding ciphertext -- $\mathrm{C}_{\mathrm{i}-1}$ :

$$
\begin{gathered}
C_{i}=P_{i} \operatorname{XORE}\left(K, C_{i-1}\right) \quad C_{0}=I V \\
P_{i}=E\left(K, C_{i-1}\right) \text { XOR } C_{i}
\end{gathered}
$$

-Duplicate plaintext blocks (patterns) NOT exposed
-Block rearrangement is detectable
-Key-stream is dependent on plaintext
-How does that affect speed of encryption? Parallelism?
-Bit error in one ciphertext block $\rightarrow$ one-bit + one-block loss in plaintext

- Adversary can still selectively flip/change bits
-One-block ciphertext loss $\boldsymbol{\rightarrow}$ 1-extra-block loss
-Can encrypt less than block size


## CTR Mode

## Counter (CTR) Mode

-Key-stream is produced by encryption increasing counter:

$$
\begin{gathered}
C_{i}=E(K, C T R) \text { XOR } P_{i} \quad C T R++ \\
P_{i}=E(K, C T R) \text { XOR } C_{i}
\end{gathered}
$$

-Duplicate plaintext blocks (patterns) NOT exposed, unless?
-Block rearrangement is detectable
-Key-stream is independent of plaintext
-Parallel encryption and decryption (random access)
-Bit error in one ciphertext block $\rightarrow$ one-bit error in plaintext
$\bullet$ One-block ciphertext loss $\rightarrow$ big mess
-Can encrypt less than block size

## MAC Mode

## Message Authentication Code (MAC) Mode

-Encryption is the same as in CBC mode, but, ciphertext is NOT sent!

$$
C_{i}=E\left(K, P_{i} \text { XOR }_{i-1}\right) \quad C_{0}=I V
$$

What is sent or stored: $P_{1}, \ldots, P_{n}, C_{n}=$ MAC

Receiver recomputes $\mathrm{C}_{\mathrm{n}}$ with K and compares
-Any change in plaintext results in unpredictable changes in MAC

## How to strengthen DES: the case of double DES

- 2DES: C = DES (K1, DES (K2, P ) )
- Seems to be hard to break by "brute force", approx. $2^{111}$ trials
- Assume Eve is trying to break 2DES and has a single ( $P, C$ ) pair

Meet-in-the-middle ATTACK:
I. For each possible $\mathrm{K}_{\mathrm{i}}^{\prime}$ (where $0<\mathrm{i}<2^{56}$ )

1. Compute $\mathrm{C}_{\mathrm{i}}^{\prime}=\mathrm{DES}\left(\mathrm{K}_{\mathrm{i}}, \mathrm{P}\right)$
2. Store: $\left[\mathrm{C}^{\prime}{ }_{i}, \mathrm{~K}_{\mathrm{i}}{ }_{\mathrm{i}}\right.$ ] in look-up table T (indexed by $\mathrm{C}^{\prime}{ }_{\mathrm{i}}$ )
II. For each possible $\mathrm{K}^{\prime \prime}{ }_{\mathrm{i}}$ (where $0<\mathrm{i}<2^{56}$ )
3. Compute $\mathrm{C}^{\prime \prime}{ }_{i}=\mathrm{DES}^{-1}\left(\mathrm{~K}_{\mathrm{i}}, \mathrm{C}\right)$
4. Look up $\mathrm{C}^{\prime \prime}$ in T
5. If lookup succeeds, output: $\mathrm{K} 1=\mathrm{K}^{\prime}{ }_{\mathrm{i}}, \mathrm{K} 2=\mathrm{K}^{\prime \prime}{ }_{i}$

## DES Variants

○ 2-DES:
$\bigcirc C=E(K 2, E(K 1, P)) \rightarrow 57$ effective key bits (meet-in-the-middle attack)

- 3-DES (Triple DES)
$\bigcirc C=E(K 3, D(K 2, E(K 1, P))) \rightarrow 112$ effective key bits (meet-in-the-middle attack)
$\bigcirc C=E(K 1, D(K 2, E(K 1, P))) \rightarrow<=80$ effective key bits
- DESX
$\bigcirc C=$ K3 XOR E(K2, (K1 XOR P) ) $\rightarrow$ seems like 184 key bits
○Effective key bits $\rightarrow$ approx. 118
$\bigcirc$ Another simple variation:
$\bigcirc \mathrm{C}=\mathrm{K} 2 \mathrm{XOR} \mathrm{E}(\mathrm{K} 1, \mathrm{P}) \rightarrow$ weak!


## DES Variants

## Why does 3-DES (or generally n-DES) work?

## Because, as a function, DES is not a group...

A "group" is an algebraic structure. One of its properties is that, taking any 2 elements of the group (a,b) and applying an operator F() yields another element c in the group.

Suppose: C = DES(K1,DES(K2,P))
There is no $K$, such that:
for each possible plaintext $P, \operatorname{DES}(\mathrm{~K}, \mathrm{P})=\mathrm{C}$

## DES Summary

- Feistel network based block cipher
- 64-bit data blocks
- 56-bit keys (8 parity bits)
- 16 rounds (shifts, XORs)
- Key schedule
- S-box selection secret ...
- DES "aging"
- 2-DES: meet-in-the-middle attack
-3-DES: 112-bit security
- DESX: 118-bit security


# Advanced Encryption Standard (AES): The Rijndael Block Cipher 

## Introduction and History

- National Institute of Science and Technology (NIST) regulates standardization in the US
- By mid-90s, DES was an aging standard that no longer met the needs for strong commercial-grade encryption
- Triple-DES: Endorsed by NIST as a "de facto" standard
- But ... slow in software and large footprint (code size)
- Advanced Encryption Standard (AES)
- Goal is to define the Federal Information Processing Standard (FIPS) by selecting a new encryption algorithm suitable for encrypting (non-classified non-military) government documents
- Candidate algorithms must be:
- Symmetric-key ciphers supporting 128, 192, and 256 bit keys
- Royalty-Free
- Unclassified (i.e., public domain)
- Available for worldwide export
- 1997: NIST publishes request for proposal
- 1998-1999: 15 submissions -> 5 finalists
- 2000: NIST chooses Rijndael as AES


## Introduction and History

- AES Round-3 Finalist Algorithms (ranked by vote \# in AES Round-2, high to low):
- Rijndael
- by Joan Daemen and Vincent Rijmen (Belgium)
- Serpent
- by Ross Anderson (UK), Eli Biham (ISR) and Lars Knudsen (NO)
- Twofish
- From Counterpane Internet Security, Inc. (MN)
- RC6
- By Ron Rivest of MIT \& RSA Labs, creator of the widely used RC4/RC5 algorithm and "R" in RSA
- MARS
- Candidate offering from IBM Research


## Rijndael

## The Winner: Rijndael

- Joan Daemen (of Proton World International) and Vincent Rijmen (of Katholieke Universiteit Leuven).
- Pronounced "Rhine-doll"
- Allows only 128,192 , and 256 -bit key sizes (unlike other candidates)
- Variable input block length: 128, 192, or 256 bits. All nine combinations of key-block length possible.
- A block is the smallest data size the algorithm will encrypt
- Vast speed improvement over DES in both hw and sw implementations
- 8,416 bytes/sec on a 20 MHz 8051
- 8.8 Mbytes/sec on a 200 MHz Pentium Pro


## Rijndael



Encryption Rounds $r_{1} \ldots r_{n}$

- Key is expanded to a set of $n$ round keys
- Input block P put thru n rounds, each with a distinct round sub-key.
- Strength of algorithm relies on difficulty of obtaining intermediate results (or state) of round i from round i+1 without the round key.

Rijndael


## Detailed view of round $n$

- Each round performs the following operations:
- Non-linear Layer: No linear relationship between the input and output of a round
- Linear Mixing Layer: Guarantees high diffusion over multiple rounds
- Very small correlation between bytes of the round input and the bytes of the output
- Key Addition Layer: Bytes of the input are simply XOR’ed with the expanded round key


## Rijndael

- Three layers provide strength against known types of cryptographic attacks: Rijndael provides "full diffusion" after only two rounds
- Cryptanalysis
- Key recovery attack:
- Best one only 4x faster than exhaustive search [BKR'11]
- Related key attack:
- AES-256: Given 2^99 input/output pairs from 4 related keys in AES256 can recover keys in time 2^99 [BK'09]
- However, how realistic is that?


## Rijndael: ByteSub



Each byte at the input of a round undergoes a non-linear byte substitution according to the following transform:

$$
\left[\begin{array}{l}
y_{0} \\
y_{1} \\
y_{2} \\
y_{3} \\
y_{4} \\
y_{5} \\
y_{6} \\
y_{7}
\end{array}\right]=\left[\begin{array}{llllllll}
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1
\end{array}\right]\left[\begin{array}{l}
x_{0} \\
x_{1} \\
x_{2} \\
x_{3} \\
x_{4} \\
x_{5} \\
x_{6} \\
x_{7}
\end{array}\right]+\left[\begin{array}{l}
1 \\
1 \\
0 \\
0 \\
0 \\
1 \\
1 \\
0
\end{array}\right]
$$

Substitution ("S")-box

## Rijndael: ShiftRow

| Nb | C 1 | C 2 | C 3 |
| :--- | :---: | :---: | :---: |
| 4 | 1 | 2 | 3 |
| 6 | 1 | 2 | 3 |
| 8 | 1 | 3 | 4 |

Depending on the block length, each "row" of the block is cyclically shifted according to the above table


## Rijndael: MixColumn



Each column is multiplied by a fixed polynomial

$$
C(x)=\prime 03^{\prime *} X^{3}+{ }^{\prime} 01^{\prime *} X^{2}+\prime 01^{\prime} * X+2^{\prime}
$$

This corresponds to matrix multiplication $b(x)=c(x) \otimes a(x)$ :

$$
\left[\begin{array}{l}
b_{0} \\
b_{1} \\
b_{2} \\
b_{3}
\end{array}\right]=\left[\begin{array}{llll}
02 & 03 & 01 & 01 \\
01 & 02 & 03 & 01 \\
01 & 01 & 02 & 03 \\
03 & 01 & 01 & 02
\end{array}\right]\left[\begin{array}{l}
a_{0} \\
a_{1} \\
a_{2} \\
a_{3}
\end{array}\right]
$$

## Rijndael: Key Expansion and Addition

| $a_{0,0}$ | $a_{0,1}$ | $a_{0,2}$ | $a_{0,3}$ | $a_{0,4}$ | $a_{0,5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $a_{1,0}$ | $a_{1,1}$ | $a_{1,2}$ | $a_{1,3}$ | $a_{1,4}$ | $a_{1,5}$ |
| $a_{2,0}$ | $a_{2,1}$ | $a_{2,2}$ | $a_{2,3}$ | $a_{2,4}$ | $a_{2,5}$ |
| $a_{3,0}$ | $a_{3,1}$ | $a_{3,2}$ | $a_{3,3}$ | $a_{3,4}$ | $a_{3,5}$ |

$\oplus$

| $k_{0,0}$ | $k_{0,1}$ | $k_{0,2}$ | $k_{0,3}$ | $k_{0,4}$ | $k_{0,5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $k_{1,0}$ | $k_{1,1}$ | $k_{1,2}$ | $k_{1,3}$ | $k_{1,4}$ | $k_{1,5}$ |
| $k_{2,0}$ | $k_{2,1}$ | $k_{2,2}$ | $k_{2,3}$ | $k_{2,4}$ | $k_{2,5}$ |
| $k_{3,0}$ | $k_{3,1}$ | $k_{3,2}$ | $k_{3,3}$ | $k_{3,4}$ | $k_{3,5}$ |


$=$| $b_{0,0}$ | $b_{0,1}$ | $b_{0,2}$ | $b_{0,3}$ | $b_{0,4}$ | $b_{0,5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $b_{1,0}$ | $b_{1,1}$ | $b_{1,2}$ | $b_{1,3}$ | $b_{1,4}$ | $b_{1,5}$ |
| $b_{2,0}$ | $b_{2,1}$ | $b_{2,2}$ | $b_{2,3}$ | $b_{2,4}$ | $b_{2,5}$ |
| $b_{3,0}$ | $b_{3,1}$ | $b_{3,2}$ | $b_{3,3}$ | $b_{3,4}$ | $b_{3,5}$ |

Each word is simply XOR'ed with the expanded round key

## Key Expansion algorithm:

```
KeyExpansion(int* Key[4*Nk], int* EKey[Nb*(Nr+1)])
{
```

```
    for(i = O; i < Nk; i++)
```

    for(i = O; i < Nk; i++)
        EKey[i] = (Key[4*i],Key[4*i+1],Key[4*i+2],Key[4*i+3]);
        EKey[i] = (Key[4*i],Key[4*i+1],Key[4*i+2],Key[4*i+3]);
    for(i = Nk; i < Nb * (Nr + 1); i++)
    for(i = Nk; i < Nb * (Nr + 1); i++)
    {
    {
        temp = EKey[i - 1];
        temp = EKey[i - 1];
        if (i % Nk == 0)
        if (i % Nk == 0)
        temp = SubByte(RotByte(temp)) ^ Rcon[i / Nk];
        temp = SubByte(RotByte(temp)) ^ Rcon[i / Nk];
        EKey[i] = EKey[i - Nk] ^ temp;
        EKey[i] = EKey[i - Nk] ^ temp;
    }
    }
    }

```
```

