Rina Dechter, Fall-01

ICS 275a, Assignment 3

Chapter 3

1. (question 6 chapter 3) Prove that the minimal network is always path-consistent: Any consistent pair of values can always be extended to any third variable.

2. (question 7 chapter 3, extra credit) Prove that a bi-valued non-empty path-consistent network is consistent. Prove that it is also the minimal network.

3. Consider 2-cnf formulas (conjunction of clauses of length 2).

   (a) Describe an algorithm for enforcing arc-consistency of 2-cnf formulas using resolution.

   (b) Describe an algorithm for enforcing path-consistency on 2-cnf formulas, using resolution.

4. (question 10 chapter 3)

   (a) Let $\mathcal{R}$ be an arbitrary 3-graph coloring problem. Discuss what would be the effect of enforcing 2-consistency on $\mathcal{R}$, what would be the effect of enforcing 3-consistency, or 4-consistency on $\mathcal{R}$.

   (b) In general, what would be the effect $2, 3, 4, ...k-1, k, k+1$-consistency on a k-coloring problem?

5. (question 16, chapter 3) Generate an arc and path-consistent network which is equivalent to the network:

$$D_x: \ x \in [1, 10], \quad D_y: \ y \in [5, 15], \quad R_{xy}: \ x + y = 10$$

$$D_z: z \in [-10, 10], \quad R_{yz}: y + z \leq 3$$

Chapter 4

6. Read chapter 4 and provide a short review answering the following questions. Typos and comments on the manuscript are welcomed.

   - Is the chapter clear? readable?
   - Is the chapter technically difficlult?
   - Enough examples and illustrations?
   - Did you enjoy/suffer while reading?
   - Would you prefer more proofs to be included? less proofs?

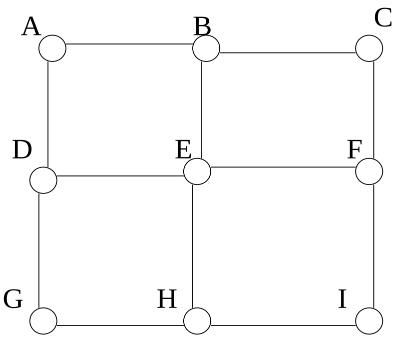


Figure 1: Grid with nine nodes

- Please provide an overall evaluation

7. (question 3 chapter 4) Let (G^*, d) be an ordered induced graph. Prove that G^* is chordal.

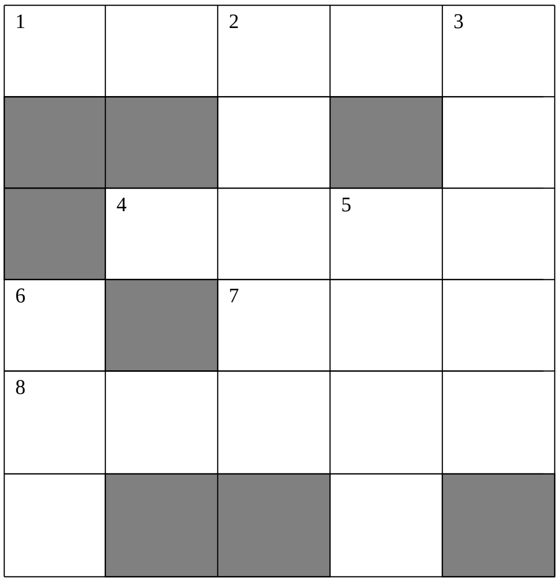
8. (question 7 chapter 4) Consider the graph in Figure 1.

   (a) What is the induced-width of the graph? Provide an ordering having minimum induced-width.

   (b) Assume that the graph expresses a binary constraint network with some constraints (e.g., inequalities). Provide a complexity bound using the induced-width for applying algorithm DPC along the optimal induced-width ordering of this problem.

   (c) Can algorithm DPC always decide consistency of every constraint problem having the grid constraint graph? Either prove or give a counter example.

9. (question 12 chapter 4) Consider the crossword puzzle:



Word List

aft	laser
ale	lee
eel	line
heel	sails
hike	sheet
hoses	steer
keel	tie
knot	

(a) Model the problem as a binary csp, that is, where the words are the variables (the problem has 8 variables). Draw its constraint graph.

(b) Generate a min-induced-width and max-cardinality ordering of the constraint graph. Generate the induced graph along these orderings. What is the w* of this problem?

(c) What level of directional-i-consistency is guaranteed to generate a backtrack-free representation for the ordered graphs that you picked, no matter what are the exact constraints?

(d) Using the min-induced-width ordering, show the constraints which will be recorded for this specific problem by 1) directional arc-consistency, 2) directional path-consistency 3) adaptive-consistency

10. (optional) Consider the crypt-arithmetic problem TA+DB = GBA. and use the formulation of the problem with carries.

(a) Draw the primal constraint graph of the problem

(b) Find an ordering of the variables using the min-induced-width algorithm in Chapter 2. What is the width of the ordering you generated. Compute the induce graph and the induced-width of the ordering generated.

(c) Hand-simulate algorithm adaptive-consistency on the ordering you created First describe the schemes of the initial partitioning into buckets, then show how new relations are created. Describe only the schemes of the relations.

(d) Bound the complexity of the algorithm on the ordering of your choice.