ICS 275B
Homework Assignment 2
Spring 2005

Instructor: Rina Dechter Due: Wednesday, April 20th

1. Question 3.1, from Pearl's book
2. **(10 pt)** Refering to the directed graph in Figure 1, determine whether or not each of the following Probabilistic independencies is true using the D-separation criterion.

   (a) $I(E, G|\phi)$.
   (b) $I(C, D|\phi)$.
   (c) $I(C, D|G)$.
   (d) $I(B, C|A)$.
   (e) $I(\{C, D\}, E|\phi)$.
   (f) $I(F, \{E, H\}|A)$.
   (f) $I(\{A, C\}, \{H, E\}|D)$.

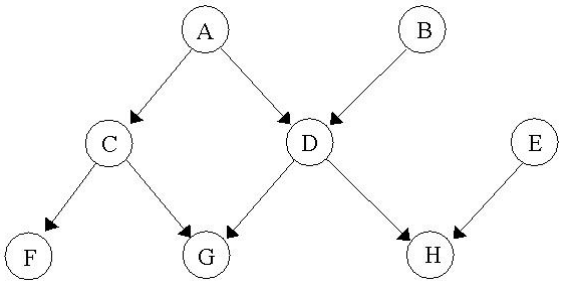

Figure 1: A directed graph.

3. **(10 pt)** Consider a set of four variables $\{X, Y, Z, W\}$, which are related by:

$$I(X, Y|\phi) \text{ and } I(X, Z|\{Y, W\}).$$

Find the minimal list of independencies generated by the above two, satisfying each of the following conditions separately.

(a) The symmetry property.
(b) The symmetry and decomposition properties.
(c) The semigraphoid properties.
(d) The graphoid properties.

4. **(10 pt)** Suppose that the joint probability distribution of four variables $\{X, Y, Z, W\}$ can be factorized as:

$$p(x, y, z, w) = p(x)p(y|x)p(z|x)p(w|y, z).$$

Determine whether or not each of the following independencies holds:

(a) $I(X, W|Y)$.
(b) $I(X, W|Z)$.
(c) $I(X, W|Y, Z)$.
(d) $I(Y, Z|X, W)$.

5. **(10 pt)** Question 3.3 a,b,d from Pearl's book

6. **(10 pt)** Question 3.6 from Pearl's book

7. **(10 pt)** Given the directed graph in Figure 1, let M be the set of independencies expressed by the dag using the d-separation criterion.

   (a) Find a Bayesian network structure (a minimal I-map) of M along the ordering: F, C, D, B, A, H, E, G.

   (b) Find the Markov network of M.

8. **(extra credit, 10 pt)** Prove that the d-separation is equivalent to regular separation in an the ancestral graph. Namely that Z d-separete X from Y if in the moral graph that includes X, Y, Z and their ancestors Z separates X from Y.

9. **(Modeling question, 30 pts)**

   In this problem you will use the software **REES** [1] OR **JAVABAYES** [2] to build a Bayesian network for "The Apprentice" TV show. We will provide you with the

[1] *available at http://www.ics.uci.edu/∼radum/*
[2] *available at http://www-2.cs.cmu.edu/∼javabayes/Home/*

nodes in the network (and their intended meanings), and it will be your job to add appropriate edges and supply values for the CPDs.

You should NOT add any extra nodes or change the name of an existing node. "The Apprentice" is a TV reality show where contestants compete to become part of one of Donald Trump's companies. The contestants are divided into two teams. Each week, the teams are given a task, which ranges from selling lemonade to designing a new line of toys. A team leader is chosen by the team (not by Donald Trump), who then manages the other members of his/her team that week (the leader also helps with the actual task). Then, each team is evaluated on their performance; the winning team gets a reward (such as eating dinner with Donald Trump!), while the losing team is sent to the board room. There, Donald Trump decides, based on individual performance and the performance of the team leader, who to eliminate from the game - "You're fired." You are a contestant on "The Apprentice", and you want to know what the probability is that you will get fired this week. For concreteness, suppose that each team currently has 5 members (including you), and that the task for this week is to sell official Donald Trump wigs - "The most luxurious wig on the planet." Each person gets 10 wigs which they want to sell for as much as possible.

For simplicity, and in order to keep the number of parameters small, monetary amounts have been restricted to a small number of possible values. You have decided on the following nodes (and their meanings):

(a) Expertise - Your expertise and/or experience in selling similar products. Takes values {High,Low}.

(b) Leadership - Your personal leadership ability. This represents your general leadership ability, not your leadership ability with respect to this week's task. Takes values {High,Low}.

(c) Leader? - Whether or not you are chosen to be the team leader for this week. Takes values {Yes,No}.

(d) Leader Performance - How well your team leader performs this week. This only measures his/her performance as a leader, not as a salesperson. Takes values {Good,Bad}.

(e) Task Difficulty - How difficult it is to sell Donald Trump wigs. Takes values {Easy, Hard}.

(f) Your Sales - How much money you (personally) make selling wigs. Takes values {Little($100),Average($10K),Lots($100K)}.

(g) Team member Sales - How much money your team members made, not including you. Takes values {Little($500),Average($50K),Lots($500K)}.

(h) Team Sales - How much money your team made, including you. Takes values {Little(Under $25K),Average($25K-$200K),Lots(Over $200K)}.

(i) Other Team Sales - How much the other team made. Takes same values as Team Sales.

(j) Deserve Fired? - Whether you deserve to be fired as compared to your team members, as judged by an "impartial" observer based on performance this week. In other words, if one of your team members must be fired, are you the one who (most) deserves to be fired? Takes values {Yes,No}.

(k) Team Wins? - Whether your team wins. Takes values {Yes,No}.

(l) Random DT? - Donald Trump decides to fire someone from the losing team for a reason unrelated to this week's task. Takes values {Yes,No}.

(m) You're Fired! - You're Fired! Takes values {Yes,No}.

If both teams end up in the same money "bracket", the tie is broken randomly. Keep in mind that you should only add edges which represent direct influence - a fully connected network is not an acceptable answer. We assume that a priori the teams are approximately equal, as are the contestants (including you). Thus, $P(TeamWins = Yes) = 0.5$, and $P(DeserveFired? = Yes) = 0.2$.

The following inequalities should also hold in your completed network (here, >> and << denote "significantly larger" and "significantly smaller", respectively):

- $P(TeamWins? = Yes|Expertise = High) > P(TeamWins? = Yes)$.
- $P(TeamWins? = Yes|Leadership = High) > P(TeamWins? = Yes)$.
- $P(You'reFired! = Yes|Leadership = High) < P(You'reFired! = Yes)$.
- $P(Leadership = High|You'reFired! = No) > P(Leadership = High)$.
- $P(You'reFired! = Yes|OtherTeamSales = Little) < P(You'reFired! = Yes)$.
- $P(RandomDT? = Yes|You'reFired! = Yes, YourSales = Lots, LeaderPerformance = Good) > P(RandomDT? = Yes)$
- $P(RandomDT? = Yes|You'reFired! = No, YourSales = Little, LeaderPerformance = Bad, Leader? = Yes) > P(RandomDT? = Yes)$
- $P(OtherTeamSales = Little|Leader? = Yes, LeaderPerformance = Bad, You'reFired! = No, YourSales = Little, RandomDT? = No) >> P(OtherTeamSales = Little)$
- $P(Leader? = Yes|YourSales = Average, OtherTeamSales = Little, RandomDT? = No, You'reFired! = Yes) >> P(Leader? = Yes)$.
- $P(DeserveFired? = Yes|You'reFired! = Yes) >> P(DeserveFired? = Yes|You'reFired! = Yes, RandomDT? = Yes)$

You should be able to satisfy each of these inequalities using "reasonable" parameters.

TO SUBMIT:

To submit your network, save your network in appropriate format (in JAVABAYES you can save the network in XMLBIF or BIF format while in REES you can save your network in "mod" format using the SAVE option) and email it to *vgogate@ics.uci.edu* with the following subject **"APPRENTICE FILE: 275B"**. Also include what software was used to create the network in the body of the message.