# COMPSCI 276
# Homework Assignment 6
Spring 2017

Instructor: Rina Dechter Due: Wednesday, May 24th

**Relevant reading:** Class notes (Dechter chapter 6-7 and chapter 8) and Darwiche chapter 15.

1. (5) Please provides comments and typos for chapter 8 (provided in the course schedule).
2. This question investigates the AND/OR search space of the network in Figure 1, assuming each variable has 2 values in its domain. The CPTs are given in Figure 2. The CPTs for $G$, $H$ and $D$ are identical to the 3-dimensional CPT in Figure 2 and the CPTs for $C$ and $F$ are identical to the two-dimensional CPT in that figure.

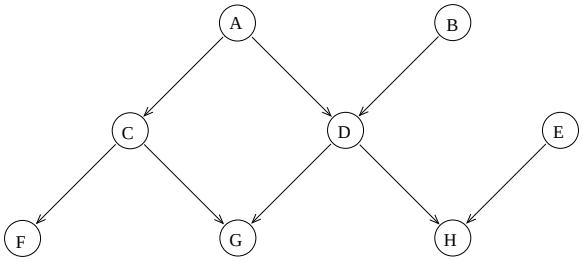


Figure 1: A directed graph

   (a) (5) Find and present a pseudo tree of the network in Figure 1 whose depth is minimal (do the best you can). Call this tree $T_1$.

   (b) (10) Generate an AND/OR search tree driven by $T_1$ assuming each variable has at most two values.

   (c) (5) Annotate the arcs with the appropriate weights.

   (d) (10) What would be the computational cost of computing the probability of evidence $G = 0$ and $H = 1$ in such a network if you use depth-first search over the AND/OR search tree. Demonstrate the computation (compute the value of each node)

   (e) (5) Can the AND/OR search tree be reduced to a smaller AND/OR search graph? if so, demonstrate.

   (f) (5) Compare the time and space complexity of solving this problem by search with context-based caching vs bucket-elimination.

   (g) (10) Assume that the CPT $P(x|y, z)$ is changed by making some entries deterministic, as follows: set the probability column of the first 2 lines to 1 and 0 in that order (from top to bottom). Do the same for the last 2 lines (e.g., we make $P(x = 0|y = 1, z = 1) = 1$). Show what would be the changes in the AND/OR search tree as a result.

| $a$ | $p(a)$ |
|---|---|
| 0 | 0.3 |
| 1 | 0.7 |

| $b$ | $p(b)$ |
|---|---|
| 0 | 0.6 |
| 1 | 0.4 |

| $e$ | $p(e)$ |
|---|---|
| 0 | 0.7 |
| 1 | 0.3 |

| $y$ | $x$ | $p(x\|y)$ |
|---|---|---|
| 0 | 0 | 0.10 |
| 0 | 1 | 0.90 |
| 1 | 0 | 0.30 |
| 1 | 1 | 0.70 |

| $z$ | $y$ | $x$ | $p(x\|y,z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.25 |
| 0 | 0 | 1 | 0.75 |
| 0 | 1 | 0 | 0.60 |
| 0 | 1 | 1 | 0.40 |
| 1 | 0 | 0 | 0.10 |
| 1 | 0 | 1 | 0.90 |
| 1 | 1 | 0 | 0.20 |
| 1 | 1 | 1 | 0.80 |

Figure 2: Conditional probability tables

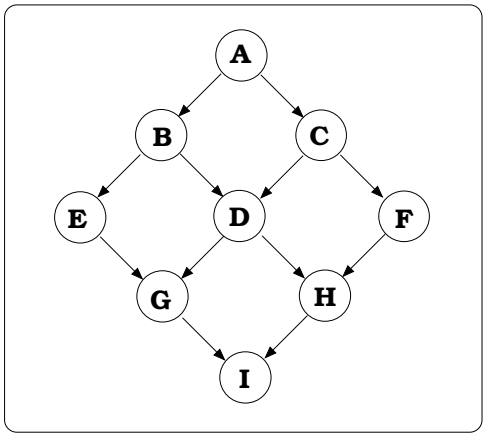


Figure 3: A Bayesian network

3. Consider the Bayes network DAG in Figure 3:

   (a) (10) Discuss the performance of the bucket-elimination algorithm for finding the belief of $P(A|I = 0)$. Demonstrate its performance schematically (describe algebraically what function is computed in each bucket). What would be the complexity of the algorithm?

   (b) (10) Apply the approximation algorithm mbe-bel(i=3) for the task of finding the belief in $A$. Trace the algorithm's performance schematically (show functions, no numbers). What is the time and space complexity of the algorithm?

   (c) Apply the weighted mini-bucket algorithm wmbe-bel(i=3) with uniform weights for the task of finding the belief in $A$. Trace the algorithm's performance schematically (show functions, no numbers). What is the time and space complexity of the algorithm?

   (d) (10) Apply mbe-mpe(i=3) to find an upper bound for the mpe of the network given evidence $I = 0$. Trace the algorithms. Show how you construct an approximate mpe tuple.

   (e) (10) Propose a node duplication simplification for the network that will correspond to the mini-bucket scheme you used in your answer above.

4. (20) Consider the coding networks in the class notes in chapter 8 (figure 8.7),

(a) What is the relaxed network, generated by variable duplication that would correspond to the mini-bucket execution of this example. Draw the relaxed network.

(b) Apply schematically, mini-clustering to find the belief of each variable. Show the tree-decomposition over which the mini-clustering algorithm executes.