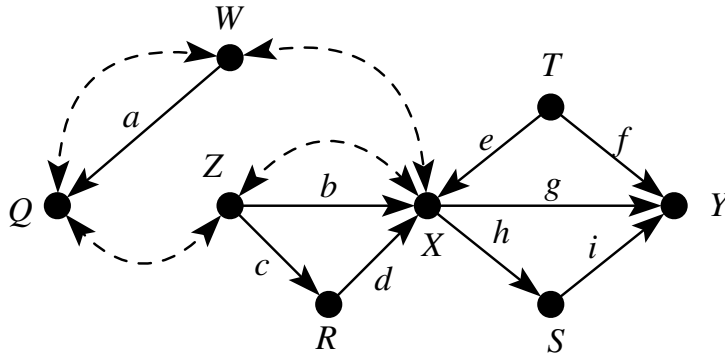


Causal and Probabilistic Graphical Models
ICS 276 (Winter 2024, Rina Dechter)
HOMEWORK 4

Due: Wednesday, March 6th, 2024

Problem 1. Linear Models [25 points]

Consider the following (linear) causal diagram below:



The lowercase letters next to each edge represent the corresponding structural coefficients.

- (a) [5 points] Assume we perform a linear regression following the equation

$$Y = \alpha_1 X + \alpha_2 R + \alpha_3 Z + \alpha_4 Q, \tag{1}$$

where α_i , $i = 1, 2, 3, 4$ are the corresponding regression (not structural) coefficients. Is any α_i equal to 0? Explain your reasoning.

- (b) [5 points] Is $E[Y|do(X)]$ identifiable in this case? If so, explain your reasoning.
 (c) [15 points] Identify as many structural coefficients as possible. Justify (briefly) each answer.

Problem 2. Study question 4.3.2 from the Primer [10 points]

(Hint: read Primer section 4.3.4)

- (a) [5 points] Describe how the parameters a , b , c in Figure 4.1 can be estimated from nonexperimental data

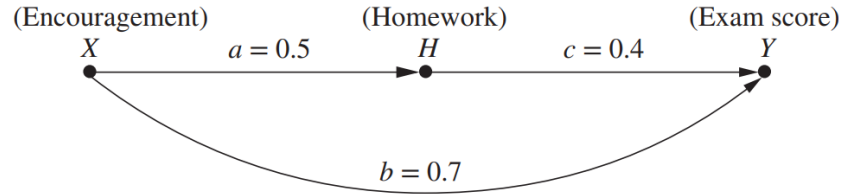


Figure 4.1 A model depicting the effect of Encouragement (X) on student's score

- (b) [5 points] In the model of Figure 4.3, find the effect of education on those students whose salary is $Y = 1$. [Hint: Use Theorem 4.3.2 to compute $E[Y_1 - Y_0 | Y = 1]$.]

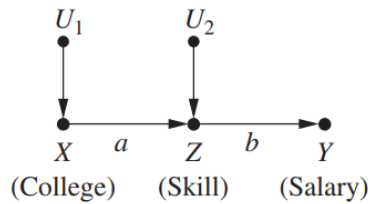
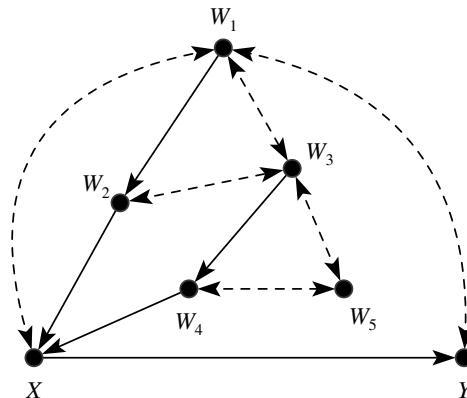


Figure 4.3 A model representing Eq. (4.7), illustrating the causal relations between college education (X), skills (Z), and salary (Y)

Problem 3. Algorithmic Identification, extra credit. [15 points]

Consider the causal diagram G shown below.



- (a) [3 points] Show how $P(y | do(x))$ decomposes in terms of c-factors.
- (b) [10 points] Use the **Identify** (see slides) algorithm to compute each one of the c-factors associated with the query from $P(\mathbf{v}) = Q[\mathbf{V}]$ as well as the effect $P(y | do(x))$. Write down the trace of the algorithm.

- (c) [2 points] Derive the same causal effect ($P(y \mid do(x))$) using the do-calculus. (You are allowed to use the tool Fusion (url: <https://causal-fusion.net>) to check your answer.)