Causal and Probabilistic Reasoning

Slides Set 8:

Introduction to Causality

Rina Dechter

(Causal Inference in Statistics, A primer, J. Pearl, M Glymur and N. Jewell Ch1, Why, ch1

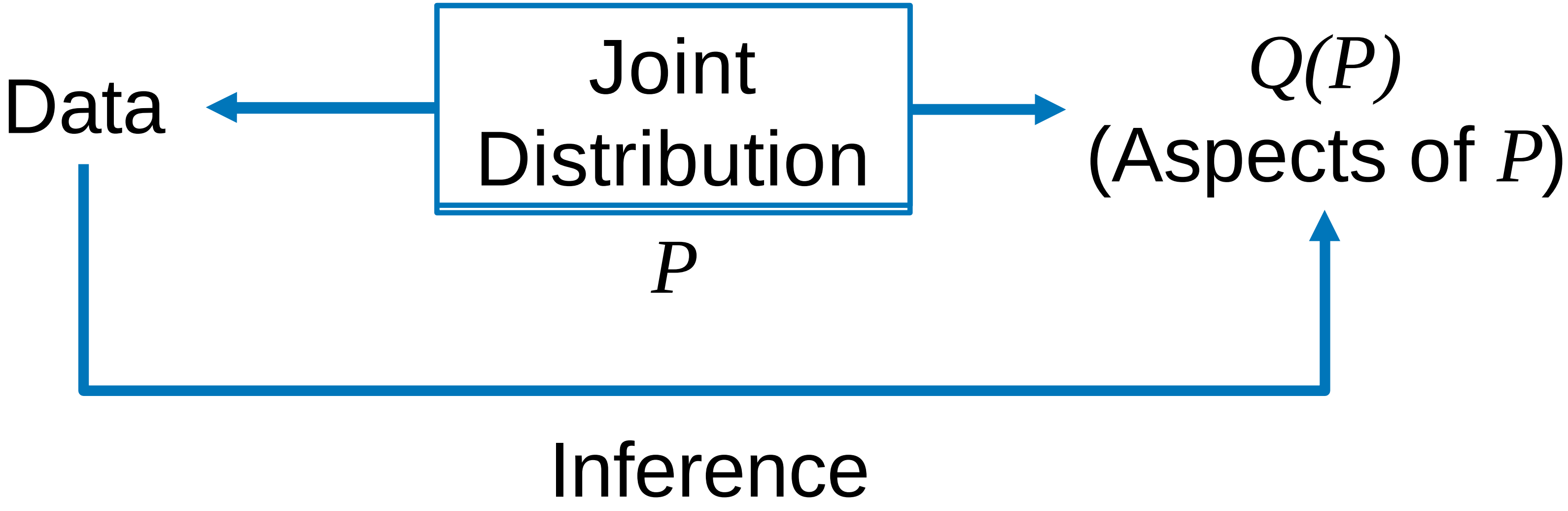
Slides by Elias Bareinboim

Outline

- Structural Causal Models (continued)
- Product form of Markov SCM
- d-separation (refresher)

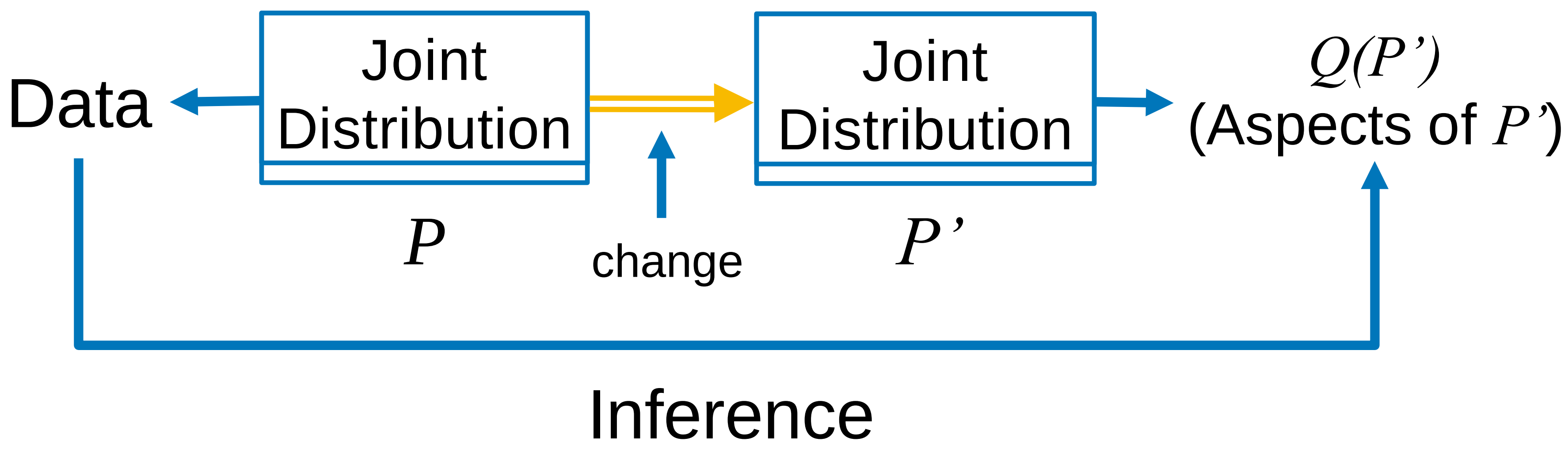
Traditional Stats-ML Inferential Paradigm

- **Approach:** Find a good representation for the data.



e.g., Infer whether customers who bought product A would also buy product B — or, compute $Q = P(B \mid A)$.

From Statistical to Causal Analysis

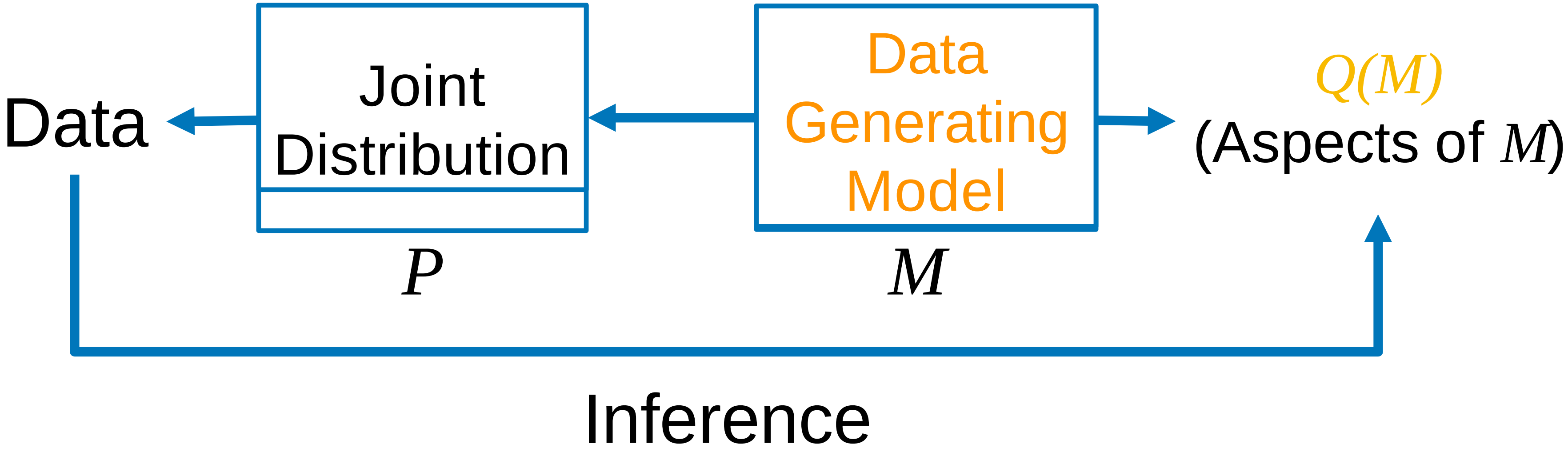


e.g., Estimate *P'(sales)* if we double the price
Estimate *P'(cancer)* if we ban smoking

Q: How does *P* (factual) changes to *P'(hypothetical)*?

Needed: New formalism to represent both P & P'.
P is tied to the data; P' is never observed, no data.

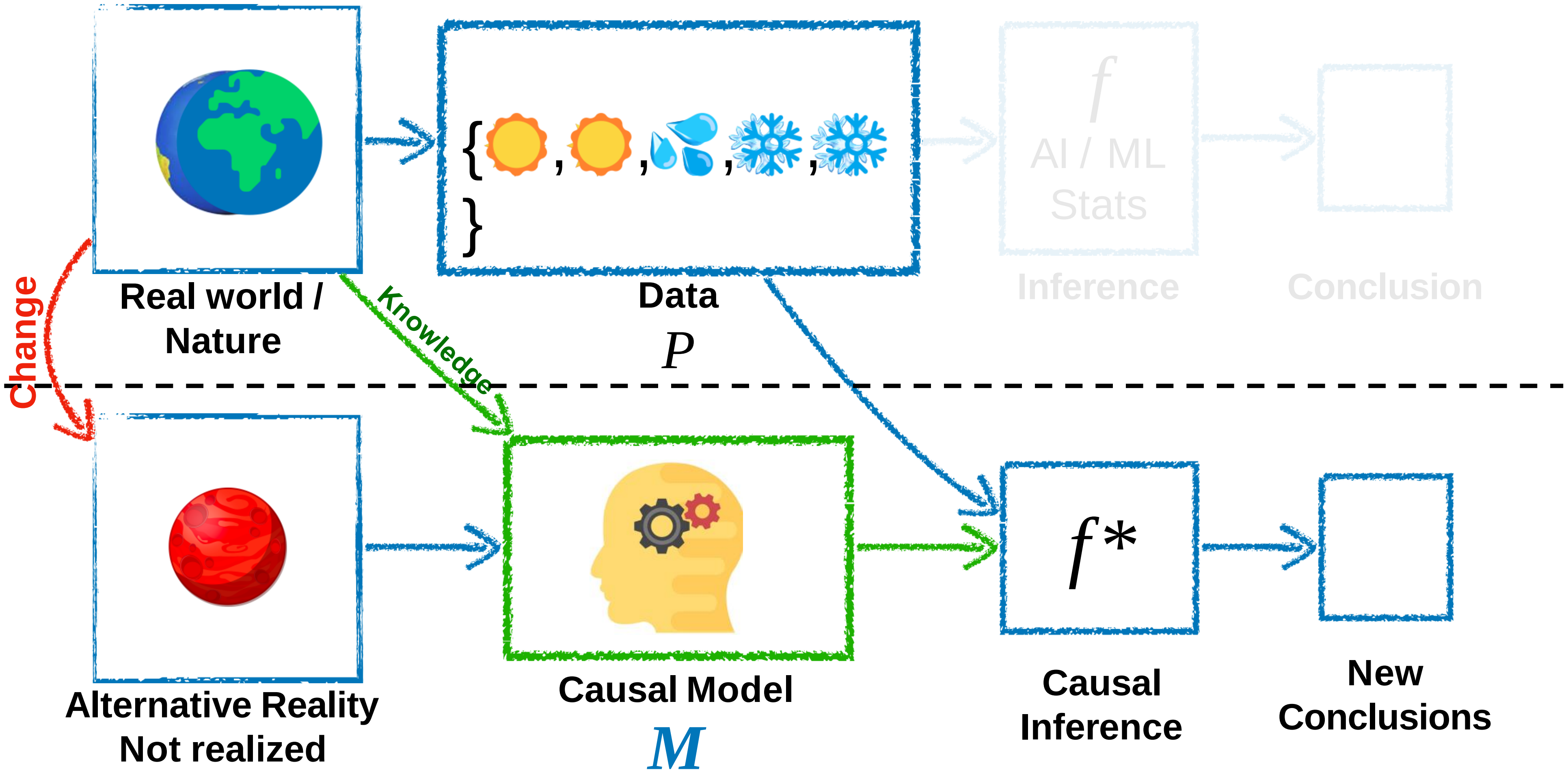
New Oracle - The Structural Causal Model Paradigm



M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

P - model of data, M - model of reality

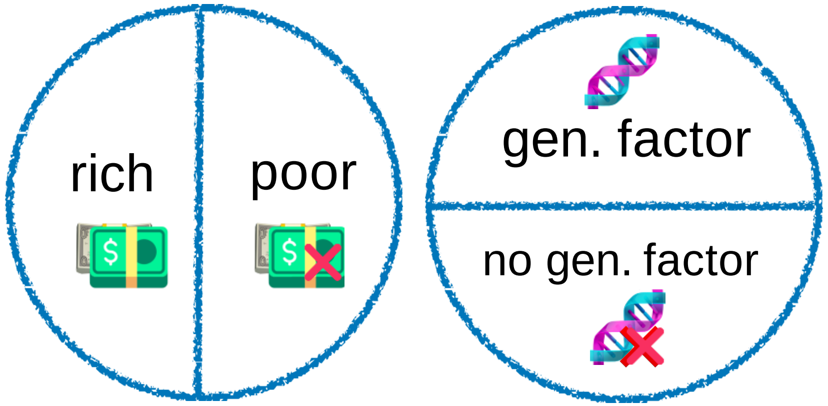
Back to the Big Picture



Modeling Reality with SCM

- The population of a certain city is falling ill from a contagious disease. There is a drug believed to help patients survive the infection.
- Unknown to the physicians, folks with good living conditions (rich) will always survive.
- While some people have a gene that naturally fights the disease and don't require treatment, they will develop an allergic reaction if treated, which is fatal under poor living conditions.

population structure



Being rich and having the genetic factor are independent events.

<u>Reality (unknown to physicians):</u>

rich = alive anyways
$poor_1$ = die anyways (no gene)
$poor_2$ = die iff take the drug (gene)

$\prod = rich \cup poor_1 \cup poor_2$

$P(rich) = P(poor)$
$P(poor_1) = P(poor_2)$

Modeling Reality in our Example

Variables we observe (**V**):

R ($R=1$ for rich, $=0$ for poor)

D ($D=1$ for taking the drug)

A ($A=1$ if person ends up alive)

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Variables that are unobserved (**U**):

U_g ($U_g=1$ has genetic factor, $=0$ o/w)
U_r (Other factors affecting Wealth)

Modeling Reality in Our Example

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How are the observed variables determined?

$R \leftarrow U_r$
$D \leftarrow R$
$A \leftarrow R \vee (U_g \wedge \neg D)$

Variables that are unobserved (**U**):

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Variables that are unobserved (**U**):

U_g ($U_g=1$ has genetic factor, $=0$ o/w)
U_r (Other factors affecting Wealth)

- Rich is always alive.
- Poor will survive only if they have the gene and don't take the drug.

Modeling Reality in our Example

Variables we observe (**V**):

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D ($D=1$ for taking the drug)

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Variables that are unobserved (**U**):

U_g ($U_g=1$ has genetic factor, $=0$ o/w)
U_r (Other factors affecting Wealth)

- What is the randomness over the unobserved vars:
- $P(U_g=1)=1/2$, $P(U_r=1)=1/2$

Modeling Reality in our Example

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How are the observed variables determined?

$R = U_r$

$A = R$

What is the randomness over the hidden variab

$P(G=1)=1/2$, $P(U_r=1)=$

2

This is a fully specified Model of Reality!

It implies both P and P' (more soon).

This will be our new, almighty Oracle,
which is known as **Structural Causal Model**.

(Now, let's generalize this object…)

Outline

- Structural Causal Models
- Product form of Markov SCM
- d-separation reresher

The New Oracle: Structural Causal Models

Definition: A structural causal model (SCM) M is a 4-tuple $<\boldsymbol{V}, \boldsymbol{U}, \mathcal{F}, P(\boldsymbol{u})>$, where

- $\boldsymbol{V} = \{V_1,...,V_n\}$ are endogenous variables;
- $\boldsymbol{U} = \{U_1,...,U_m\}$ are exogenous variables;
- $\mathcal{F} = \{f_1,..., f_n\}$ are functions determining $\boldsymbol{V}$,
  $v_i \leftarrow f_i(pa_i, u_i)$, $Pa_i \subset V_i, U_i \subset U$;
- $P(\boldsymbol{u})$ is a distribution over $\boldsymbol{U}$

Not regression!!

e.g. $y = \alpha + \beta X + U_Y$

Axiomatic Characterization:

(Galles-Pearl, 1998; Halpern, 1998).

1. SCM induces distribution $P(\mathbf{v})$

- $\mathcal{F}$ can be seen as a mapping from $\boldsymbol{U} \longrightarrow \boldsymbol{V}$

$$(u_1, u_2, \ldots, u_k) \longrightarrow \boxed{\mathcal{F}} \longrightarrow (v_1, v_2, \ldots, v_n)$$

- When the input $\boldsymbol{U}$ is a set of random vars, then the output $\boldsymbol{V}$ also becomes a set of r.v's.

- P(v) is the layer 1 of the PCH, known as the observational (or passive) prob. distribution.

- Each event, person, observation, etc… corresponds to an instantiation of $\boldsymbol{U}=\boldsymbol{u}$.

1. SCM induces distribution $\boldsymbol{P(v)}$

Example: (Drug, Rich, Alive)

- Each citizen follows in one of four groups according to the unobservables in the model:

$$\mathcal{F} = \begin{cases} f_R : U_r \\ f_D : R \\ f_A : R \vee (U_g \wedge \neg D) \end{cases}$$

$$\mathcal{F}$$

$$(U_r{=}1, U_g{=}1) \longrightarrow (R{=}1, D{=}1, A{=}1)$$

$$(U_r{=}1, U_g{=}0) \longrightarrow (R{=}1, D{=}1, A{=}1)$$

$$(U_r{=}0, U_g{=}1) \longrightarrow (R{=}0, D{=}0, A{=}1)$$

$$(U_r{=}0, U_g{=}0) \longrightarrow (R{=}0, D{=}0, A{=}0)$$

1. SCM induces distribution $P(\mathbf{v})$

In our example:

- Events in the $\boldsymbol{U}$-space translate into events in the space of $\boldsymbol{V}$.

$$\mathcal{F} = \begin{cases} f_R : U_r \\ f_D : R \\ f_A : R \vee (U_g \wedge \neg D) \end{cases}$$

$P(\mathbf{u})$			$P(\mathbf{v})$
1/4	$(U_r=1, U_g=1)$	$\longrightarrow (R=1, D=1, A=1)$	1/2
1/4	$(U_r=1, U_g=0)$	$\longrightarrow (R=1, D=1, A=1)$	
1/4	$(U_r=0, U_g=1)$	$\longrightarrow (R=0, D=0, A=1)$	1/4
1/4	$(U_r=0, U_g=0)$	$\longrightarrow (R=0, D=0, A=0)$	1/4

1. SCM induces distribution $P(\mathbf{v})$

- [Def. 2, PCH chapter] An SCM $M = <\boldsymbol{V}, \boldsymbol{U}, \mathscr{F}, P(\boldsymbol{u})>$ defines a joint probability distribution $P^M(V)$ *s.t. for each* $Y \subseteq V$:

$$P^M(y) = \sum_{u \mid Y(u)=y} P(u)$$

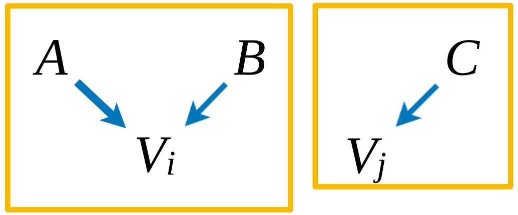
- $\mathscr{F}$ can be seen as a mapping from $\boldsymbol{U} \longrightarrow \boldsymbol{V}$

$$(u_1,u_2,\ldots,u_k) \longrightarrow \boxed{\mathcal{F}} \longrightarrow (v_1,v_2,\ldots,v_n)$$

2. SCM → Causal Diagram

- Every SCM M induces a causal diagram
- Represented as a DAG where:
  - Each $V_i \in \boldsymbol{V}$ is a node,
  - There is $W \longrightarrow V_i$ if for $W \in Pa_i$,

$$V_i \leftarrow f_i(A,B,U)$$
$$V_j \leftarrow f_j(C,U)$$



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  - There is $V_i \leftarrow\cdots\cdots\rightarrow V_j$ whenever $U_i \cap U_j \neq \varnothing$.

$V_i \leftarrow f_i(A,B,U)$
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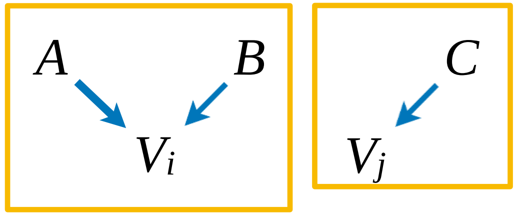

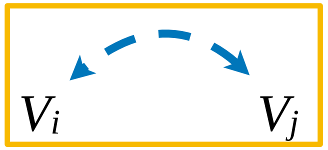

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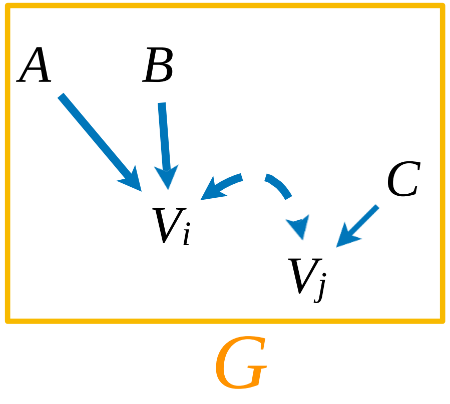


Causal Diagram — Definition (formal)

- ***Causal Diagram*** [Def. 13, PCH chapter] — Consider an SCM M = $<\boldsymbol{V}, \boldsymbol{U}, \mathcal{F}, P(\boldsymbol{u})>$. Then G is said to be a causal diagram (of M) if constructed as follows:

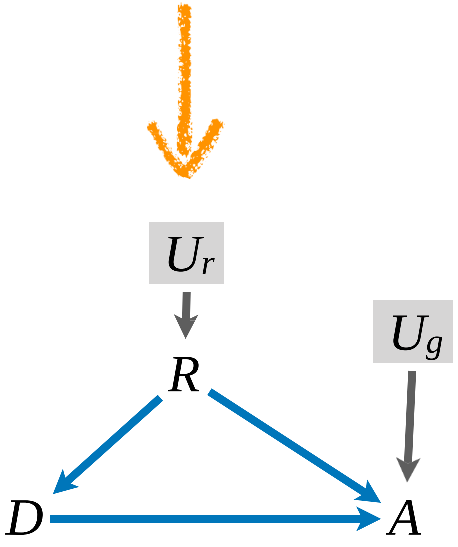
1. add vertex for every endogenous variable $\boldsymbol{V_i} \in \boldsymbol{V}$.
2. add edge ($\boldsymbol{V_j} \rightarrow \boldsymbol{V_i}$) for every $\boldsymbol{V_i}$, $\boldsymbol{V_j} \subset$V if $\boldsymbol{V_j}$ appears as argument of $\boldsymbol{f_i} \in \mathcal{F}$.
3. add a bidirected edge ($\boldsymbol{V_j} \leftarrow\!\!\!\dashrightarrow \boldsymbol{V_i}$) for every $\boldsymbol{V_i}$, $\boldsymbol{V_j} \subset$V if $\boldsymbol{U_i}$, $\boldsymbol{U_j} \subset$***U*** are correlated or the corresponding functions $\boldsymbol{f_i}$, $\boldsymbol{f_j}$ share some $U \in$**U** as argument.

2. SCM → Causal Diagram

Recall our medical example:

- Endogenous (observed) variables $\boldsymbol{V}$:
  - R ($R=1$ for rich, $=0$ for poor)
  - D ($D=1$ for taking the drug, $D=0$ o/w)
  - A ($A=1$ if person ends up alive, $=0$ o/w)
- Exogenous (unobserved) Variables $\boldsymbol{U}$:
  - U_r (Wealthiness factors)
  - U_g ($=1$ has the genetic factor, $=0$ o/w)
- Distribution over $\boldsymbol{U}$: $P(U_r)=1/2$, $P(U_g)=1/2$

$$\mathcal{F} = \begin{cases} R \leftarrow U_r \\ D \leftarrow R \\ A \leftarrow R \vee (U_g \wedge \neg D) \end{cases}$$

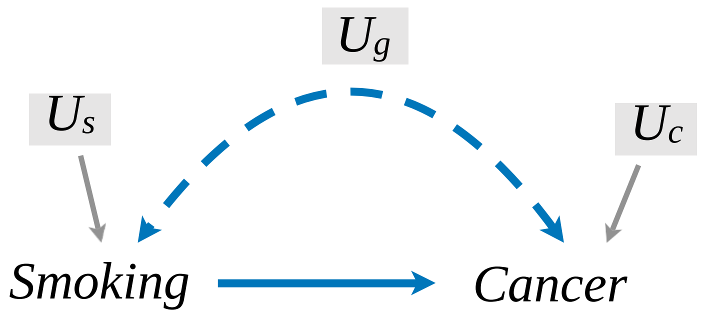


2. SCM → Causal Diagram

Another example:

- $\boldsymbol{V} = \{\ Smoking, Cancer\ \}$
- $\boldsymbol{U} = \{\ U_s, U_c, U_g\ \}$ unobserved factors
  - $\mathcal{F}$:

$$Smoking \leftarrow \mathrm{f}_{\mathrm{Smoking}}(U_s, U_g)$$
$$Cancer \leftarrow \mathrm{f}_{\mathrm{Cancer}}(Smoking, U_c, U_g)$$

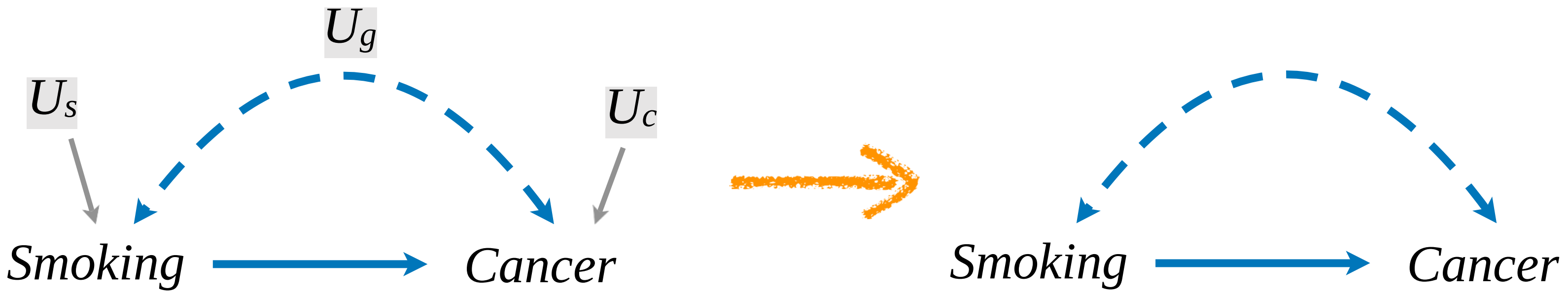


Remark 1. The mapping is just 1-way (i.e., from a SCM to a causal graph) since the graph itself is compatible with infinitely many SCMs with the same scope (the same functions signatures and compatible exogenous distributions).

Remark 2. This observation will be central to causal inference since, in most practical settings, researchers may know the scope of the functions, for example, but not the details about the underlying mechanisms.

Causal Diagrams

- Convention. The unobserved variables are left implicit in the graph.



What about this example

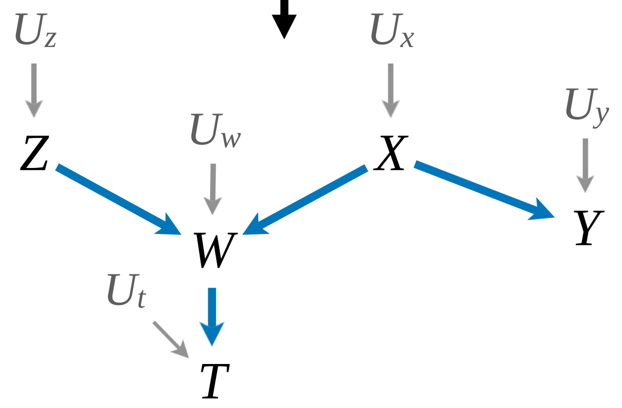
Does the causal diagram give us any clues about the (in)dependence relations in the obs. distribution P(V)?

- Is T independent of W?
- Is W independent of T?
- Is Z independent of T?
- Is Z independent of X?
- Is Y independent of W?
- Is Y independent of W if we know the value of X?

$$M = \begin{cases} Z \leftarrow f_Z(u_z) \\ X \leftarrow f_x(u_x) \\ W \leftarrow f_W(z, x, u_w) \\ Y \leftarrow f_Y(x, u_y) \\ T \leftarrow f_T(w, u_t) \end{cases}$$

Prop. 1

Prop. 2



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- Structural Causal Models
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The New Oracle:
Structural Causal Models

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- $\mathbf{V} = \{V_1,...,V_n\}$ are endogenous variables;
- $\mathbf{U} = \{U_1,...,U_m\}$ are exogenous variables;
- $\mathcal{F} = \{P1,...,Pn\}$ are CPTs for V

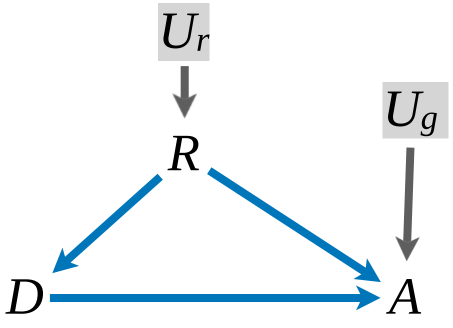
$Pa_i \subset V_i, U_i \subset U;$

- $P(\mathbf{u})$ is a distribution over $\mathbf{U}$

Pi = P(VI | pai, ui)

The Emergence of the First Layer

In our example,



The joint distribution over the observables ***P(v)*** is equal to:

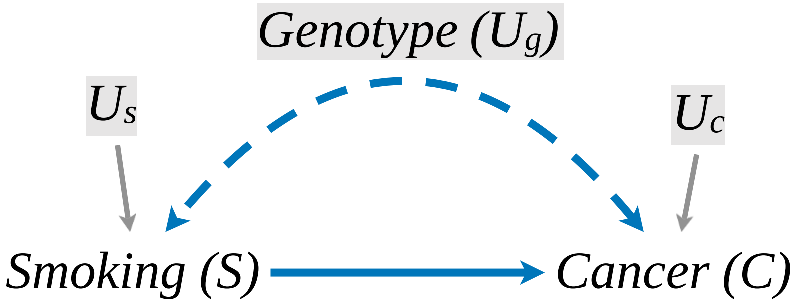
$$P(R = r, D = d, A = a) = \sum_{u_r, u_g} P(R = r, D = d, A = a, U_r = u_r, U_g = g)$$

For short,

$$P(r, d, a) = \sum_{u_r, u_g} P(r, d, a, u_r, u_g)$$

The Emergence of the First Layer

In the second example,



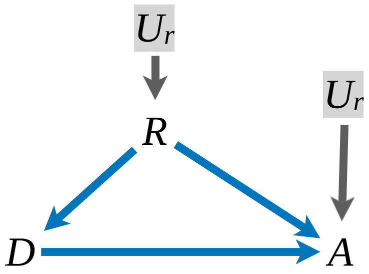
The joint probability distribution over the observed variables (V), *Smoking* and *Cancer*, is given by

$$P(s,c) = \sum_{u_s,u_g,u_c} P\left(s,c,u_s,u_g,u_c\right)$$

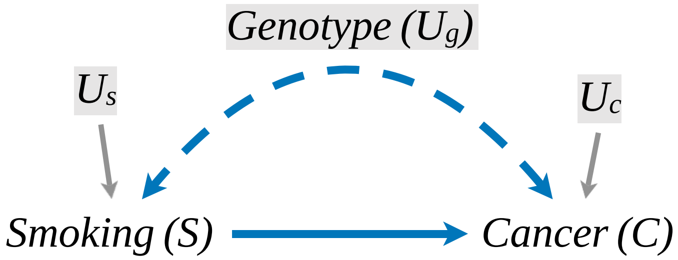
Recall, this distribution is called observational distribution. Sometimes, it's also called passive or non-experimental distribution.

What the Diagram Encodes

- Since G is a directed acyclic graph, there exists a topological order over $\boldsymbol{V}$ such that every variable goes after its parents, i.e., $Pa_i < V_i$.



$$R < D < A$$



$$S < C$$

What the Diagram Encodes

- M induces $P(\mathbf{V})$:

$$P(\mathbf{v}) = \sum_{\mathbf{u}} P(\mathbf{v}, \mathbf{u}),$$

  - Using the chain rule and following a topological order,

$$P(\mathbf{v}) = \sum_{\mathbf{u}} P(\mathbf{u}) \prod_{V_i \in \mathbf{V}} P\left(v_i \mid v_1, \ldots, v_{i-1}, \mathbf{u}\right),$$

- An observed variable is fully determined by its observed and unobserved parents; also $\{pa_i, u_i\} \subseteq \{v_1, \ldots, v_{i-1}, \mathbf{u}\}$, then

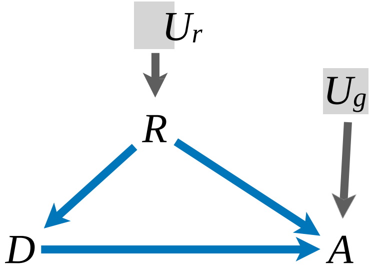
$$P(v_i \mid v_1, \ldots, v_{i-1}, \mathbf{u}) = P(v_i \mid pa_i, u_i)$$

What the Diagram Encodes

- The distribution $P(\mathbf{V})$ decomposes as:

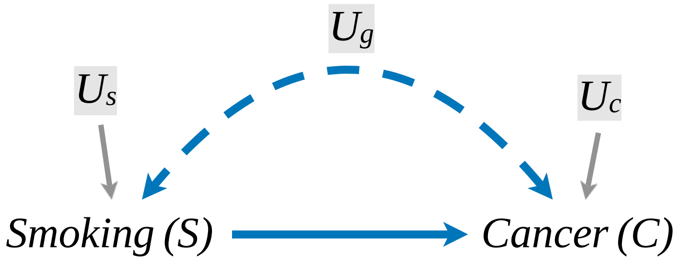
$$P(\mathbf{v}) = \sum_{\mathbf{u}} P(\mathbf{u}) \prod_{V_i \in \mathbf{V}} P\left(v_i \mid v_1, \ldots, v_{i-1}, \mathbf{u}\right)$$

$$= \sum_{\mathbf{u}} P(\mathbf{u}) \prod_{V_i \in \mathbf{V}} P\left(v_i \mid pa_i, u_i\right)$$



U
r

$$P(r, d, a) = \sum_{u_r, u_g} P(u_r, u_g) P(r \mid u_r) P(d \mid r) P(a \mid r, d, u_g)$$



$$P(s, c) = \sum_{u_s, u_g, u_c} P(u_s, u_g, u_c) P(s \mid u_g, u_s) P(c \mid s, u_g, u_c)$$

Markovian Factorization

- Suppose no variable in $\boldsymbol{U}$ is a parent of two variables in $\boldsymbol{V}$ *(observables)* (i.e., $\forall_{i,j}\ U_i \cap U_j = \emptyset$), then the model is called Markovian. We have:

$$P(\mathbf{v}) = \sum_{\mathbf{u}} P(\mathbf{u}) \prod_{V_i \in \mathbf{V}} P(v_i \mid pa_i, u_i)$$

$$= \sum_{\mathbf{u}} \prod_{V_i \in \mathbf{V}} P(v_i \mid pa_i, u_i)\, P(u_i)$$

$$= \sum_{\mathbf{u}} \prod_{V_i \in \mathbf{V}} P(v_i \mid pa_i, u_i)\, P(u_i \mid pa_i)$$

$$= \prod_{V_i \in \mathbf{V}} \sum_{u_i} P(v_i, u_i \mid pa_i) = \boxed{\prod_{V_i \in \mathbf{V}} P(v_i \mid pa_i)}$$

Bayesian Factorization

In Markovian models
SCM yields a Bayesian network
Over the visible variables

Local
Markovian
Condition

$$\boxed{(V_i \perp\!\!\!\perp Nd_i \setminus Pa_i \mid Pa_i)}$$

Causal Bayesian Networks

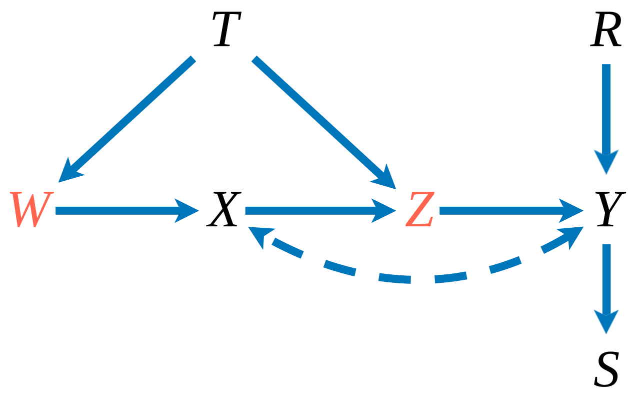
- SCM when the unctions are general CPTs.

Outline

- Structural Causal Models
- Product form of Markov SCM
- d-separation (refresher)

Graph Separation (d-Separation)

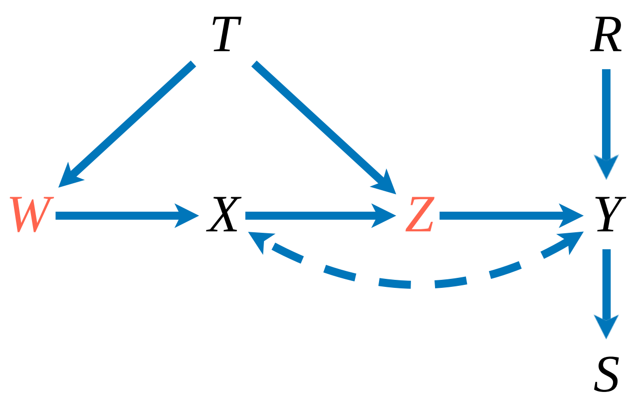
Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?



Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?

Path 1: $W \longleftarrow T \longrightarrow Z$



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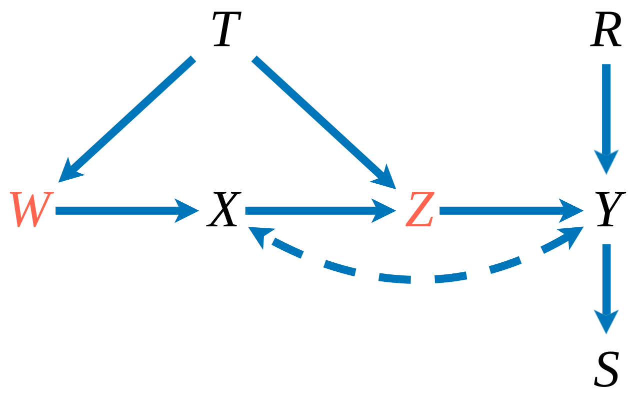

Path 1: $W \longleftarrow T \longrightarrow Z$

Path 2: $W \longrightarrow X \longrightarrow Z$

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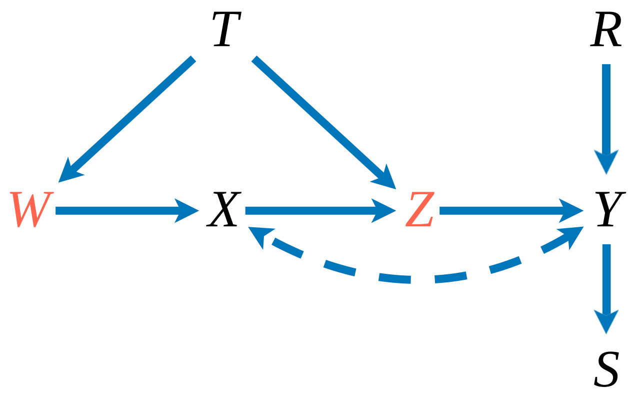


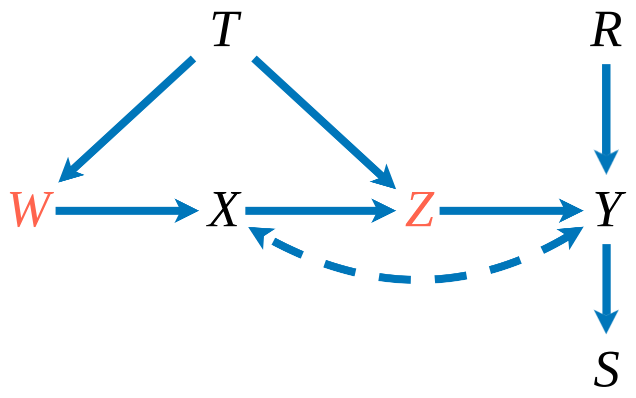
Path 1: $W \longleftarrow T \longrightarrow Z$

Path 2: $W \longrightarrow X \longrightarrow Z$

Path 3: $W \longrightarrow X \longleftrightarrow Y \longleftarrow Z$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?



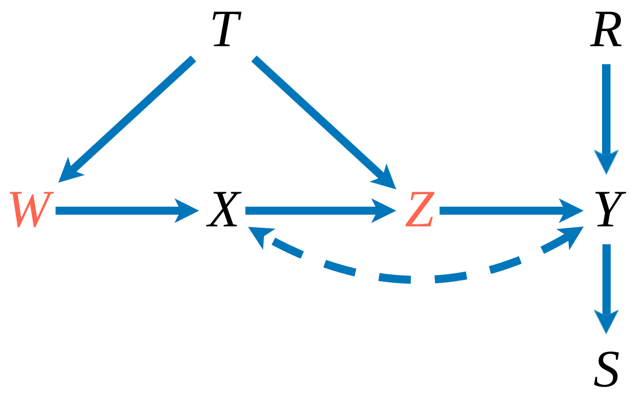
Path 1: $W \longleftarrow T \longrightarrow Z$

Path 2: $W \longrightarrow X \longrightarrow Z$

Path 3: $W \longrightarrow X \longleftrightarrow Y \longleftarrow Z$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?



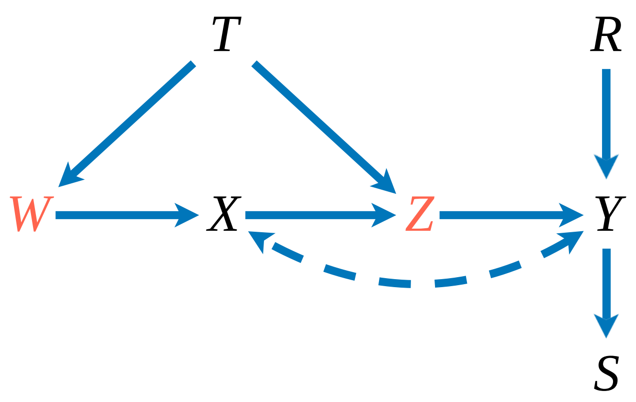
Path 1: $W \longleftarrow T \longrightarrow Z$

Path 2: $W \longrightarrow X \longrightarrow Z$

Path 3: $W \longrightarrow X \longleftrightarrow Y \longleftarrow Z$ $= W \longrightarrow X \longleftarrow U \longrightarrow Y \longleftarrow Z$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?



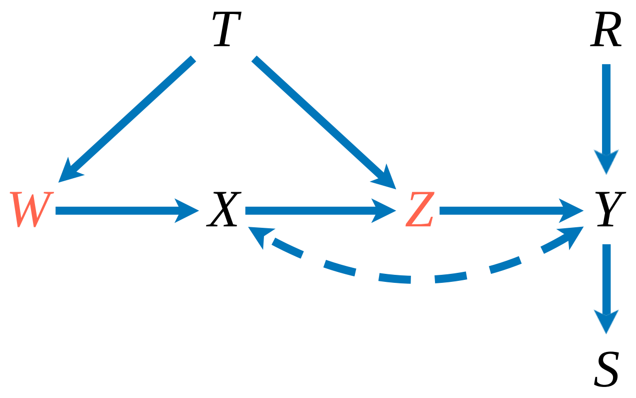
Path 1: $W \longleftarrow T \longrightarrow Z$

Path 2: $W \longrightarrow X \longrightarrow Z$

Path 3: $W \longrightarrow X \longleftrightarrow Y \longleftarrow Z \quad = W \longrightarrow X \longleftarrow U \longrightarrow Y \longleftarrow Z$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Z \mid \boldsymbol{A})$ holds?



Path 1: $W \longleftarrow T \longrightarrow Z$

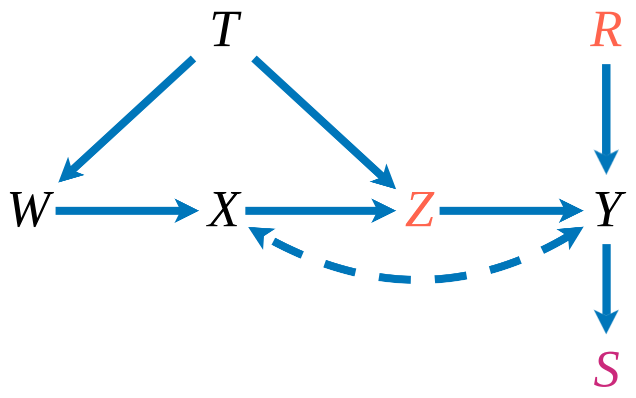
Path 2: $W \longrightarrow X \longrightarrow Z$

Path 1 and 2 need to be blocked, Path 3 is naturally blocked:
$\boldsymbol{A}=\{T, X\}$ suffices.

Path 3: $W \longrightarrow X \longleftrightarrow Y \longleftarrow Z$ $= W \longrightarrow X \longleftarrow U \longrightarrow Y \longleftarrow Z$

Graph Separation (d-Separation)

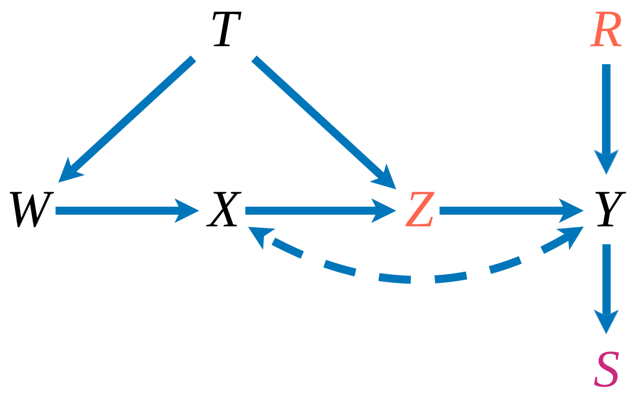
Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?



Graph Separation (d-Separation)

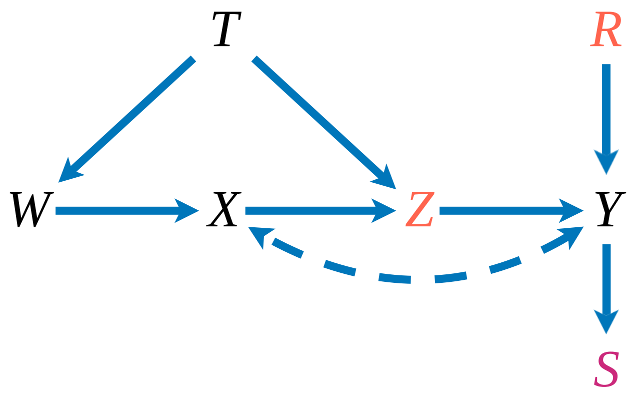
Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?

Path 1: $R \longrightarrow Y \longrightarrow S$



Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?

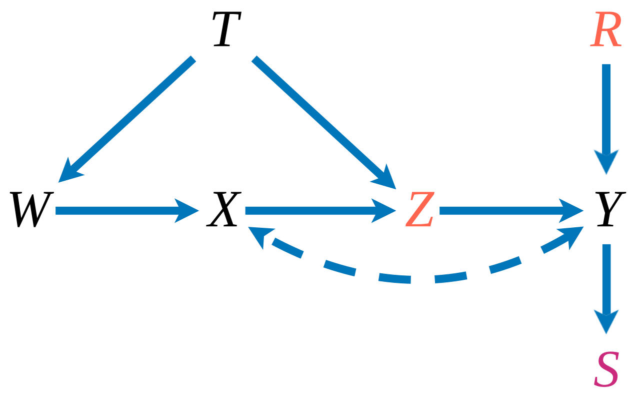


Path 1: $R \longrightarrow Y \longrightarrow S$

Path 2: $Z \longrightarrow Y \longrightarrow S$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?



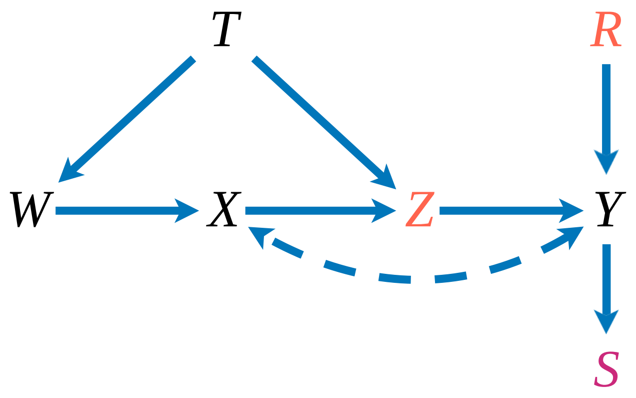
Path 1: $R \longrightarrow Y \longrightarrow S$

Path 2: $Z \longrightarrow Y \longrightarrow S$

Path 3: $Z \longleftarrow X \longleftrightarrow Y \longrightarrow S$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?



Path 1: $R \longrightarrow Y \longrightarrow S$

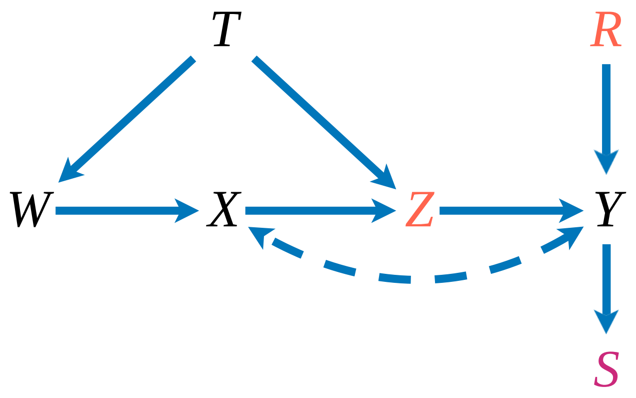
Path 2: $Z \longrightarrow Y \longrightarrow S$

Path 3: $Z \longleftarrow X \longleftrightarrow Y \longrightarrow S$

Path 4: $Z \longleftarrow T \longrightarrow W \longrightarrow X \longleftrightarrow Y \longrightarrow S$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?



Path 1: $R \longrightarrow Y \longrightarrow S$

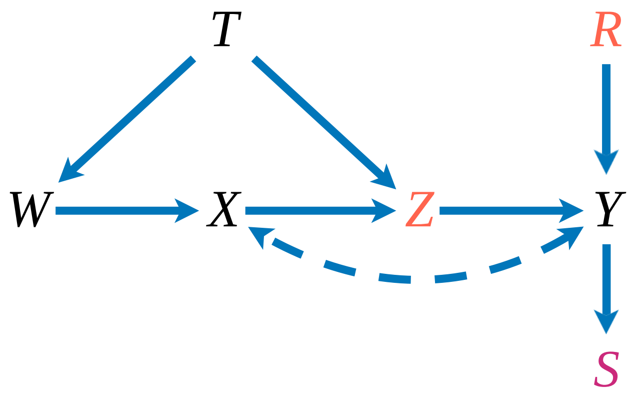
Path 2: $Z \longrightarrow Y \longrightarrow S$

Path 3: $Z \longleftarrow X \longleftrightarrow Y \longrightarrow S$

Path 4: $Z \longleftarrow T \longrightarrow W \longrightarrow X \longleftrightarrow Y \longrightarrow S$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(R, Z \perp\!\!\!\perp S \mid \boldsymbol{A})$ holds?



Path 1: $R \longrightarrow Y \longrightarrow S$

Path 2: $Z \longrightarrow Y \longrightarrow S$

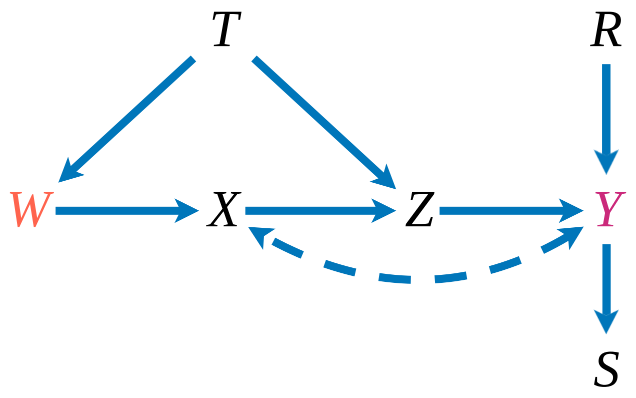
Path 3: $Z \longleftarrow X \longleftrightarrow Y \longrightarrow S$

Path 4: $Z \longleftarrow T \longrightarrow W \longrightarrow X \longleftrightarrow Y \longrightarrow S$

$\boldsymbol{A} = \{Y\}$ suffices.

Graph Separation (d-Separation)

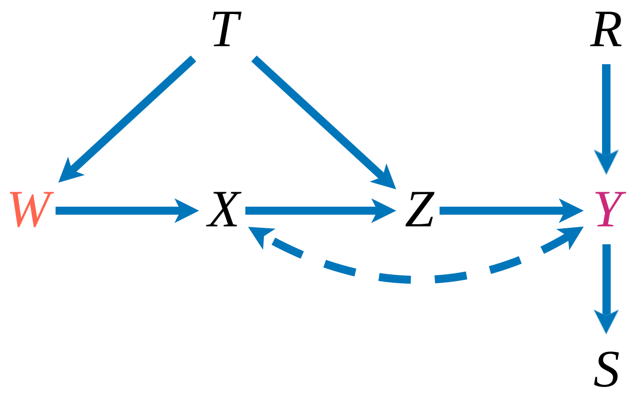
Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Graph Separation (d-Separation)

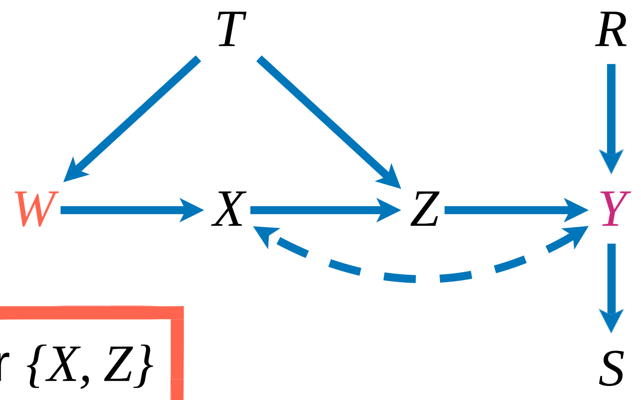
Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?

Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$



Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?

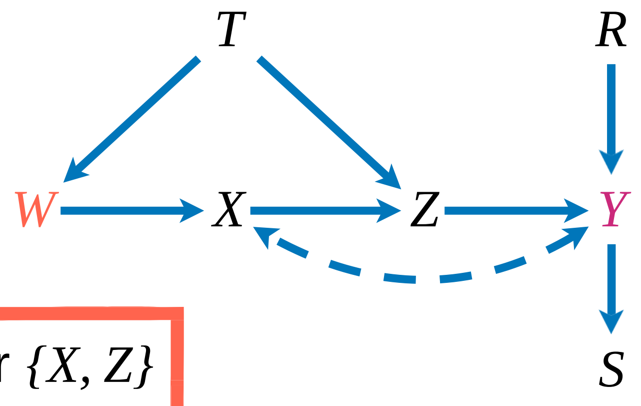


Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$

$\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



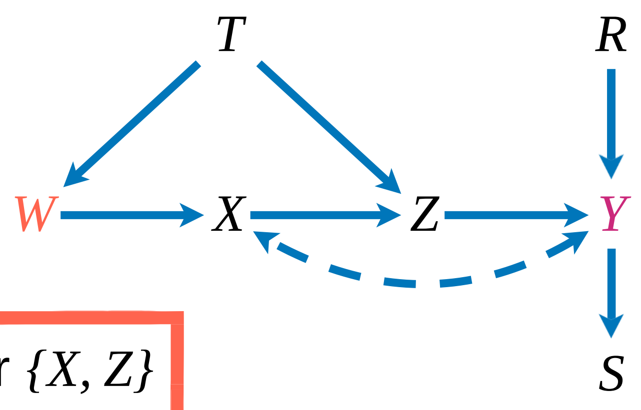
Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$

$\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?

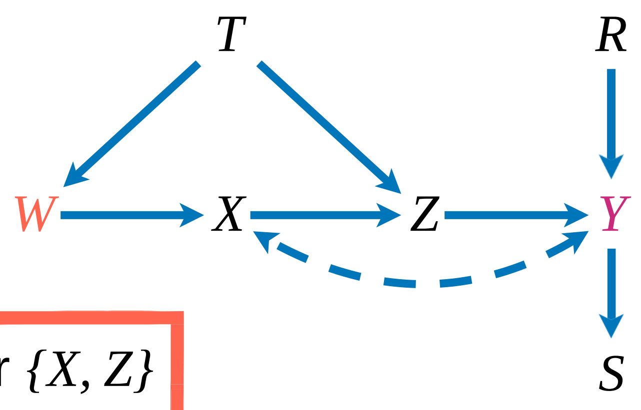


Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



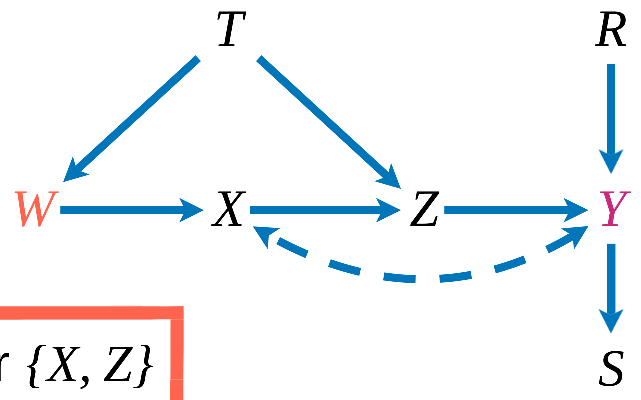
Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



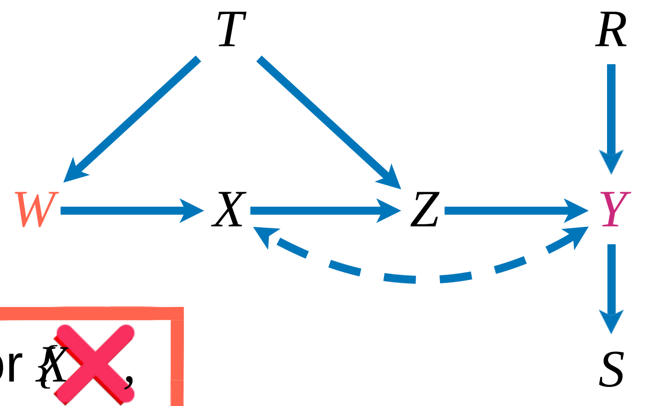
Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



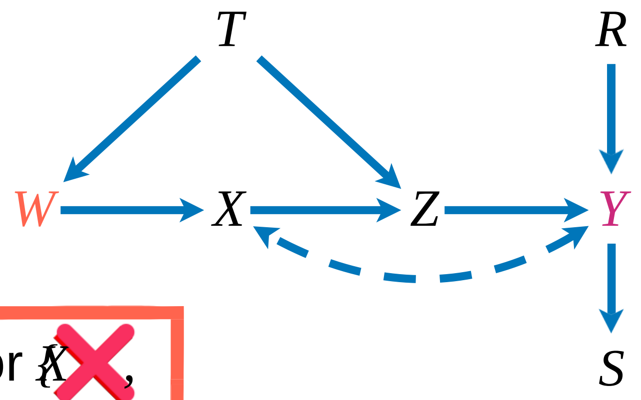
Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — ~~$\{X\}$~~ or $\{Z\}$ or ~~$\{X, Z\}$~~

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

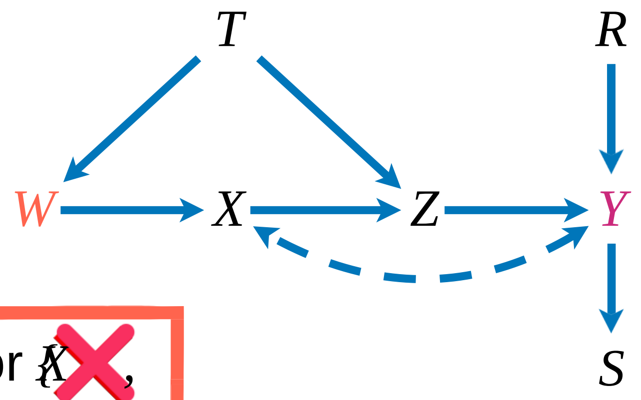
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — ~~$\{X\}$~~ or $\{Z\}$ or ~~$\{X, Z\}$~~

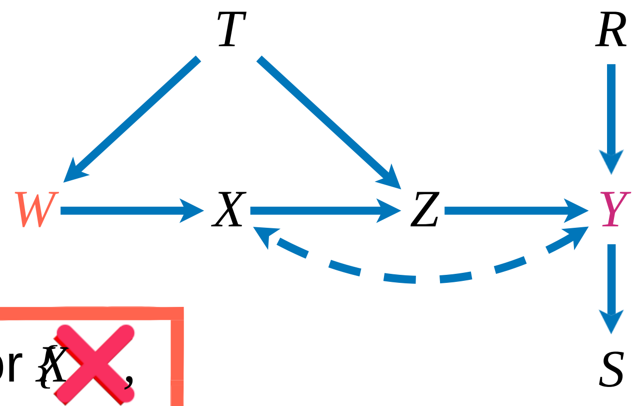
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T,X,Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

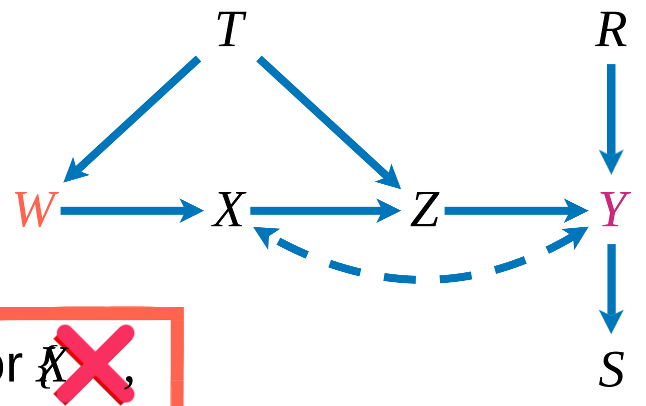
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

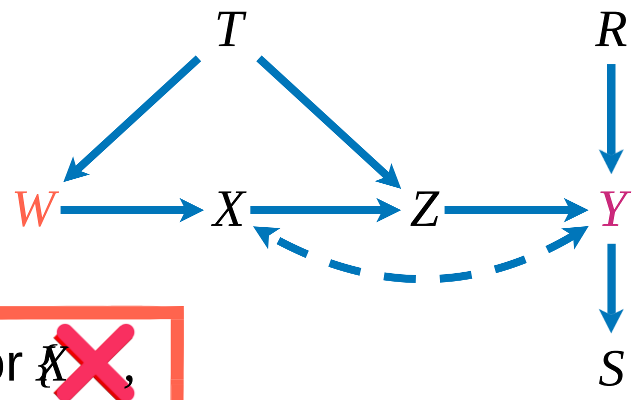
Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Does $\boldsymbol{A}=\{T, Z\}$ suffice?

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

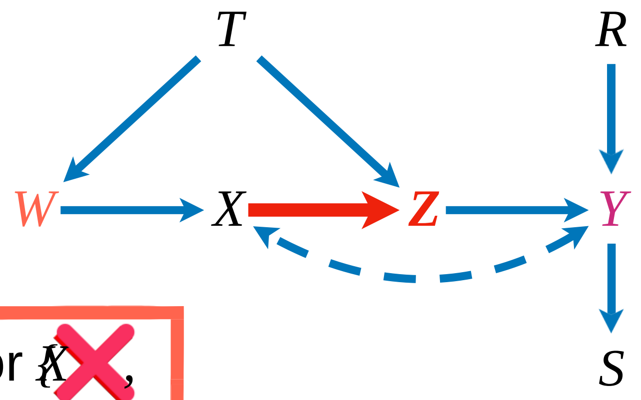
Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Does $\boldsymbol{A}=\{T, Z\}$ suffice?

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

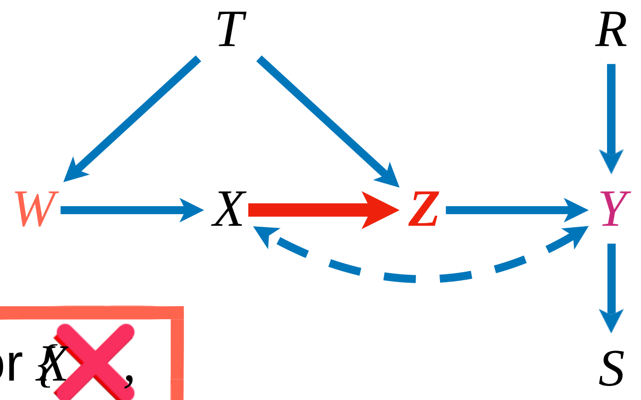
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$ — not X

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

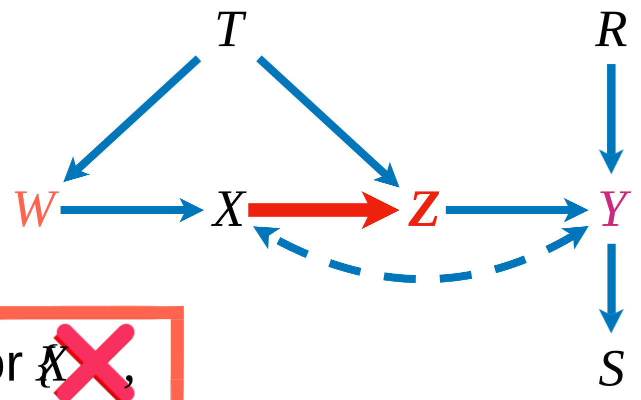
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$, $X \to Z$ — not X

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

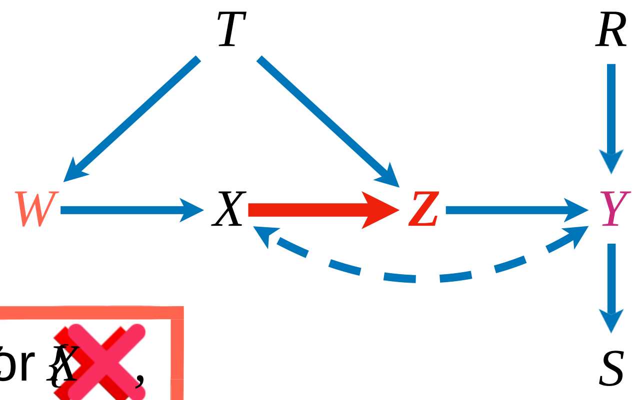
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$, $X \to Z$ — not X | not Z

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{X\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

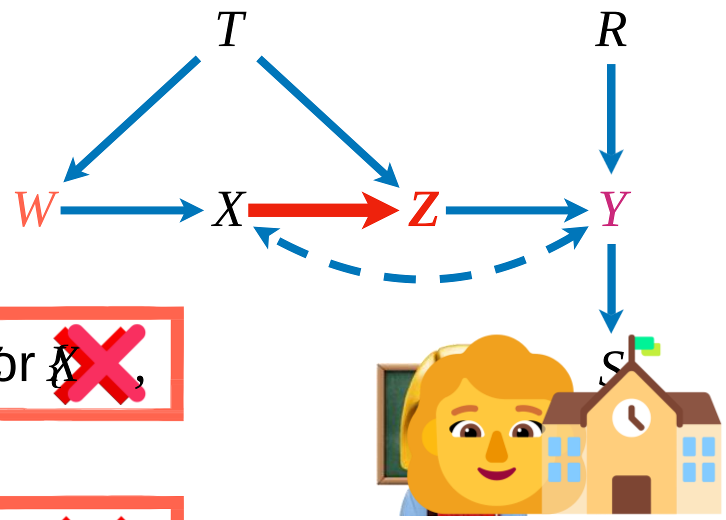
Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$, $X \longrightarrow Z$ — not X | not Z

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

Graph Separation (d-Separation)

Is there a set $\boldsymbol{A}$ such that the separation statement $(W \perp\!\!\!\perp Y \mid \boldsymbol{A})$ holds?



Path 1: $W \longrightarrow X \longrightarrow Z \longrightarrow Y$ — $\{X\}$ or $\{Z\}$ or $\{X, Z\}$

Path 2: $W \longleftarrow T \longrightarrow Z \longrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, Z\}$

Path 3: $W \longrightarrow X \longleftrightarrow Y$, $X \longrightarrow Z$ — not X — not Z

Path 4: $W \longleftarrow T \longrightarrow Z \longleftarrow X \longleftrightarrow Y$ — $\{T\}$ or $\{Z\}$ or $\{T, X\}$ or $\{T, Z\}$ or $\{T, X, Z\}$

No such $\boldsymbol{A}$!

Don't forget the descendants of the colliders!

d-SEPARATION (EXAMPLE)

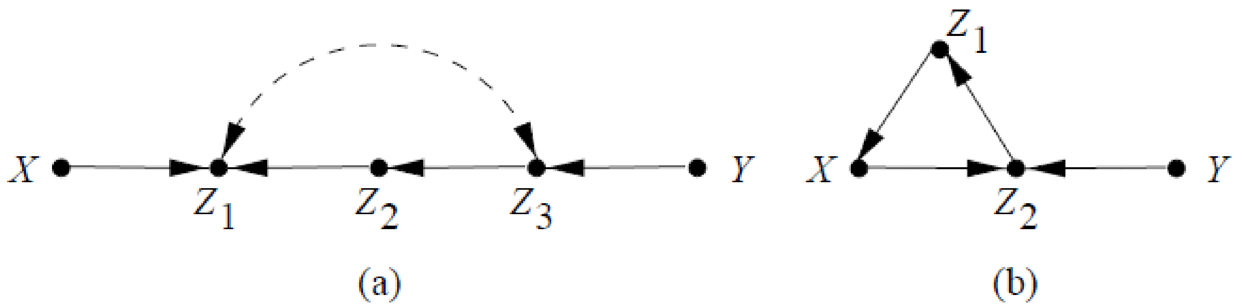


Figure 1.3: Graphs illustrating d-separation. In (a), X and Y are d-separated given Z_2 and d-connected given Z_1. In (b), X and Y cannot be d-separated by any set of nodes.